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AN INTRODUCTION TO APPLIED MACROECONOMICS

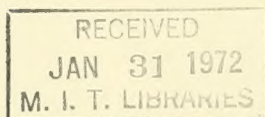
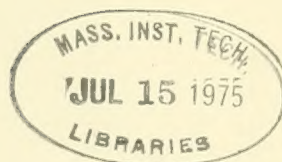
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PREFACE

We feel that any intermediate or advanced course in macroeconomics is seriously incomplete today unless it provides students with some understanding of econometric models of national economies -- macroeconometric models for short. Yet our own efforts to introduce students to econometric models of the U.S. economy have invariably been disappointing. As an outlet for these frustrations, we have written this book which is designed as a supplement to intermediate or advanced courses in macroeconomics or applied econometrics and, to a lesser extent, as a reference for researchers in the field.

Most available macroeconometric models, especially those in the journal literature, are designed to forecast Gross National Product and its components. Their primary purpose is not to illustrate the structure of the economy, but rather to follow its movement over time as closely as possible. Consequently, numerous compromises are made. Quantities which are logically endogenous variables are taken as exogenous, and many equations have only the most tenuous theoretical justification. Explanations of these models tend to be brief and oriented toward the practicing professional economist. Sources of data are rarely listed, and weaknesses in the models are rarely acknowledged.

(We do not mean to impugn the investigators involved; many of these sins of omission were obviously forced on them by journal editors or their immediate objectives.) We have tried to introduce students to macroeconomic models with Suits' [1962] article and the Office of Business Economics model (Liebenberg, et al [1966]), but for the reasons just mentioned, neither paper was adequate to the need. (References cited here are listed at the end of Chapter 1.)

At the other extreme are the large "structural" models, of which the Brookings-SSRC (Duesenberry, et al [1965], [1969]) and M.I.T. - Federal Reserve Board - University of Pennsylvania (deLeeuw and Gramlich [1968], Rasche and Shapiro [1968], and Ando and Modigliani [1969]) efforts are best known. Most equations in these models are products of careful thought and detailed investigation. Any single sector can be usefully examined at considerable length. And this is just the problem. It is difficult to assign the original Brookings volume to mortal students and have any time left in a one-semester course for other material.

Two presentations of econometric models have been designed largely for pedagogic purposes. Gregory Chow's [1967] small model is too simple; it does not provide enough insight into the structure of the economy. Michael

Evans' [1969] recent book tries to build an empirically-oriented macroeconomics course around the Wharton EFU forecasting model. We have serious reservations about this model, chiefly because it was built primarily for forecasting. Another problem is that anyone wishing to use Evans' book can hardly assign additional material, as Evans provides long, closely related discussions of macroeconomic theory and of his model. It seemed that it would be useful, therefore, to provide a model by itself, designed to complement whatever other materials the instructor wishes to assign.

Thus we believe that a large gap exists in macroeconomics texts, and this book seeks to fill that gap. We have estimated a medium-sized quarterly model of the U.S. economy designed from the outset as a teaching tool. We have tried to illustrate the state of the model building art with maximum theoretical and quantitative simplicity consistent with our view of reality.

Our aim throughout has been to use "off-the-shelf" equations, relying heavily on specifications developed by previous investigators. In large measure we have succeeded in this, and, consequently, we do not discuss all our specification decisions in detail but rather refer the interested reader to the list of references at the end of each chapter. The model is novel in a few respects,

most noticeably in its handling of fixed investment and price-wage determination; we discuss unusual specifications in more detail than the others.

Our model has numerous unsatisfactory equations, as do all macro models, and we emphasize our weaknesses. Since our avowed aim is to rely on existing knowledge, unsatisfactory equations reflect both the state of the art and our own limitations. We are not trying to sell this model to government or business; our aim is rather to stimulate interest in quantitative, applied research in macroeconomics.

Students must have some familiarity with ordinary least squares regression to understand our discussions of the estimated equations in Chapters 3-11. As there are a variety of introductory econometrics texts on the market, we do not provide a discussion of this topic. Two good introductory references are Kane [1968] and Wonnacott and Wonnacott [1970, Part I]. For a short, qualitative treatment, see Schmalensee [1972, Ch. 2].

Chapter 1 and the material in Chapter 2 on the Koyck lag should be read before going on to later chapters. The discussion of more general lag structures in Chapter 2 would also be helpful, though it is directly relevant mainly in Chapter 4. Once Chapters 1 and 2 have been covered, Chapters 3-11 can be assigned in whatever sequence

the instructor feels best suits his purposes. Chapter 12 logically follows the others.

It is a sad commentary that it is almost impossible to build a sensible model of the U.S. economy without some reliance on unpublished data. To those in and out of the federal government who furnished us with such series, we are extremely grateful. We do not list individuals' names in order to save them from an avalanche of requests, but the text does indicate the agency that supplied each set of unpublished figures. Most presentations of econometric models do not indicate data sources very carefully; we have tried to be an exception to this rule. We invite the reader to improve our specifications, and we wish him well in this task.

This model was constructed and simulated on the TROLL time-sharing system at MIT. We are grateful to the TROLL staff, headed by Mark Eisner, for helping us to use this powerful tool.

We are indebted to the Edwin Land Foundation for considerable financial support. Comments on Chapter 2 by John Hooper, on Chapters 1 and 3 by Laurence Meyer and on Chapter 10 by Richard Attiyeh and J. Phillip Cooper were most helpful. The usual disclaimer for their responsibility is hereby offered with above average force. Daniel Luria and Darhsiung Chang proved to be invaluable research assistants; this model truly could not have been built without them. Edward Hyman, Stephen Fisher, and Walter Maling also provided able assistance in the early stages. The first draft of this manuscript was typed by Yvonne Wong and Esa Rappaport, and later versions were prepared

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E.K.

R.S.

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CHAPTER 1

INTRODUCTION AND OVERVIEW1.1 PURPOSE OF THE TEXT

In the last fifteen years or so, economists have made increasing use of a tool first developed in the 1930's for analyzing the behavior of aggregate economic variables -- the macroeconometric model. Many macroeconometric models have been constructed and used to forecast such magnitudes as output, income, unemployment, and inflation, to estimate the quantitative impact of alternative government policies, and to investigate the nature of short-run macroeconomic fluctuations. As our quantitative knowledge of the economic world we inhabit grows, macroeconometric models will become even more important and useful to forecasters, policy-makers, and researchers in the years to come.

This text presents and describes the construction of a medium-sized (by today's standards) quarterly econometric model of the U.S. economy. The model can be used for forecasting and policy evaluation, as we illustrate in Chapter 12, but we have not built it primarily for these purposes. Our first aim is to convey to the reader some quantitative idea of the structure and dynamics of the U.S. economy. Second, and of equal importance, we seek to describe the

state of the art of macroeconometric modeling in a critical but constructive fashion. We are not trying to sell the reader on our model but to interest him in this area of research; this book should convince him that many important problems in quantitative macroeconomics remain unsolved.

In the remainder of this section, we discuss the general nature of econometric models. The next two sections are devoted to features of our model and its construction and to an outline of the rest of this book.

Econometrics is often described as the science (or art) of quantifying or measuring relationships suggested by economic theory. A broad definition of an econometric model, then is an economic model, usually expressed as a system of equations, with numbers that relate it to the real world. A macroeconometric model is an econometric model concerned with aggregate quantities such as GNP and the unemployment rate.

An example will serve to make these notions more precise. The simplest textbook macroeconomic model is the following:

$$\begin{aligned} C &= f(Y) \\ Y &= C + I + G, \text{ where} \\ (1.1) \quad C &= \text{Consumption Expenditure,} \end{aligned}$$

I = Investment Expenditure,

G = Government Expenditure,

Y = Gross National Product,

and $f(Y)$ is some function.

It is usually assumed that the values of I and G are determined outside the model. Variables for which this is true are termed exogenous. Given values for I and G and the function $f(Y)$, model (1.1) may be solved for C and Y . These latter quantities may thus be viewed as determined within the model; such variables are said to be endogenous. Note that when the model takes the form of a set of simultaneous equations, as (1.1) does, there must be one and only one equation corresponding to each endogenous variable.

Exogenous variables and endogenous variables determined in earlier periods are both called predetermined. If, for instance, consumption in (1.1) were assumed to be a function of current GNP and of GNP in the previous period, lagged GNP would be a predetermined variable in the model.

Model (1.1) contains equations which correspond to equations found in econometric models. Its first equation if made specific and applied to historical data would become a stochastic equation, one which does not hold exactly at all points in time. It would therefore contain an error term which is assumed to obey some stochastic i.e.,

probabilistic law. The second equation in (1.1) is an identity or accounting definition. Such equations are always exactly satisfied as a consequence of the way the variables involved in them are defined.

There are, in turn, two types of stochastic equations. The first equation in (1.1) is a behavioral equation, an attempt to describe economic actors' behavior patterns. More complicated models also contain institutional equations which attempt to summarize complex consequences of laws and other regulations. An example is the equation in Chapter 7 which estimates personal income tax payments on the basis of personal income and one tax rate. }

The first step in constructing a macroeconometric model (or any econometric model) is to formulate hypotheses about how the economy works and, thus, about what behavioral equations are likely to be stable. One might begin with model (1.1), which is based on simple Keynesian macroeconomic theory. This theory suggests the existence of a stable relationship between consumption and income. If the function $f(Y)$ is not stable, model (1.1) adds nothing to our knowledge of reality. }

Fixing the basic framework of a macroeconometric model is by no means the last decision the model-builder must make. He might decide to build a very simple aggregative model, for instance, one based directly on (1.1), containing

just a few equations, or he may decide that many more equations are needed to model features of the economy he deems important. If the aim of a model is forecasting, it might be as small as four equations; see Friend and Taubman [1964], for instance. If it is desired to capture the basic structure of a developed economy, experience suggests that twenty equations is a lower limit. Some models employ more than two hundred equations in an effort to describe the economy more exactly; see Duesenberry et al [1965, 1969] for such a large model.

Models also vary with respect to the length of the unit time period; there are monthly, quarterly, and annual models.¹ The shorter the time period chosen, the harder it is to find useable data, but the more likely the model is to capture the kinds of short-run fluctuations that interest many forecasters and policymakers.

Suppose, to be specific, that we have decided to build a quarterly model based on (1.1). We face more decisions; the detailed specification of the forms of stochastic equations always requires reference to historical data. Theory tells us, for instance that $0 < df(Y)/dY < 1$, but it does not indicate whether $f(Y)$ can best be approximated by a linear, quadratic, logarithmic, or more esoteric function. Further, in most cases the theory does not completely determine the variables that ought to be present in any

stochastic equation. Does consumption depend only on this period's GNP, for instance, or is there enough inertia in household behavior to make that last period's income also relevant?

The answers to questions of this sort can only be obtained by confronting the model with historical data. No stochastic equation ever fully explains all the historical changes in its dependent variable, as the economy is far too complex ever to model exactly. But equations compatible with the historical data are desirable, and the final form of an econometric model is determined with this criterion, as well as the relevant theory in mind.

To return to our example, one might wonder whether $f(Y)$ should be linear or quadratic. In this case one would want to investigate the compatibility of the following two alternative equations with historical data:

$$(1.2) \quad \begin{aligned} C &= a_1 + b_1 Y + u_1 \\ C &= a_2 + b_2 Y + c_2 Y^2 + u_2, \end{aligned}$$

where the a 's, b 's, and c_2 are (unknown) constants, and u_1 and u_2 are disturbance or error terms included because the rest of the equations cannot fully explain movements in C . Given historical observations on the variables C and Y , standard statistical estimation techniques can be used to provide the "best" estimates of the constants in (1.2), as

well as measures of the precision of the estimates. Given estimates of the unknown constants and observations on C and Y , estimates of the u 's for each period can be routinely calculated by subtracting the rest of the right-hand-sides of the equations from C . These estimates are often called residuals. The size of the absolute values of the u 's gives a measure of how well each equation explains movements in C .

Using standard statistical techniques, we can test the hypothesis that c_2 is zero or, equivalently, that the addition of the Y^2 term does not significantly improve the explanatory power of the second equation in (1.2). If this hypothesis cannot be rejected, and if all coefficient estimates were sensible in light of economic theory, we would select the first equation for our model on the grounds that simplicity is to be preferred in explanations of reality.

Our final model might then look like

$$(1.3) \quad \begin{aligned} C &= 20 + .6 Y \\ Y &= C + I + G \end{aligned}$$

Given forecasts for I and G , this model could be used to forecast C and Y . Alternatively, the effects of different levels of G on the economy could be predicted. Neither forecasts nor predictions from (1.3) could be expected to be exact, of course, even if the forecasts of I and G were correct, since the stochastic first equation would not

explain all changes in C .

Note that if (1.3) were to be used to forecast C or Y , both equations would be involved. Most econometric models, and this includes the one presented in this book, are systems of simultaneous equations. Thus, in general, all equations affect the forecasts (or predictions) of every individual endogenous variable.

In constructing a macroeconometric model, then, one begins with general ideas about the way the economy functions and thus about what behavioral and institutional equations are likely to be stable. One then decides on the approximate complexity (level of aggregation) of the model and on the length of the unit time period. The next step is to use historical data to answer those questions about the stochastic equations which are not answered by economic theory. In the process, one also invariably tests the original hypotheses. This selection of variables and the choice of functional forms for the stochastic equations is usually called the specification of the model.²

1.2 CONSTRUCTING THE MODEL

The model presented here has the post-Keynesian orientation of most recent macroeconometric modeling efforts. We chose to use a quarter (three months) as our unit time period in order to permit examination of short-run fluctuations and still ensure the availability of essential data.

This characteristic, too, is shared with the most recent models.

Our model is neither large as the largest efforts nor as small as the smallest. It is composed of 42 stochastic equations and 40 identities, and it thus determines 82 endogenous variables. We have tried to illustrate the state of the modeling art and the structure of the economy with minimum necessary complexity. (Of course, judgements about what level of complexity is necessary are inevitably subjective.) The model is shot through with imperfections, another feature it shares with all similar efforts.

Chapter 2 is an introduction to assumptions made about the dynamics of behavior in this model and in much of macroeconomic literature. Some familiarity with its contents is essential to the understanding of later chapters. The reader should also have some knowledge of ordinary least squares regression, the estimation method used in building this model. We do not discuss the interpretation of least squares estimates herein; the reader is referred to Schmalensee [1972, Ch. 2] for a qualitative treatment or to Kane [1968] and Wonnacott and Wonnacott [1970, Part I] for more detailed presentations.

Before outlining the rest of this book, we now discuss a number of other decisions we made in the early stages of this model's development.

It was our original intention to use only readily-available published data in our model. This proved impossible, as it has for other model-builders. We have indicated the sources of all published data and the agencies supplying unpublished figures, trying to provide more complete documentation than is standard in the presentation of macroeconometric models. We have also attempted to make our model a self-contained system, to treat only logically exogenous quantities as exogenous. This has led us to make compromises in our specification of the variables present in behavioral equations in many cases. For instance, the yield on common stocks can reasonably be expected to influence business investment decisions, but it is not currently possible to explain statistically the movement in common stock prices. Were we to use this variable in our model, we would be compelled either to tolerate a very weak equation that attempted to explain it, or to treat it as exogenous. As neither course seemed very satisfactory, this variable does not appear in our model. Instead other variables are employed to capture its effects.

Some quantities, like Inventory Valuation Adjustment, are logically endogenous and essential to the model. These we explain, but with weak equations. The need to have an equation explaining any logically endogenous variable appearing in the model is not present in studies of individual

sectors or of single variables. But it must be satisfied if an entire economy is to be properly modeled. We have tried to make our specifications as theoretically sound as possible, even in the face of this requirement, but the job was not easy.

The precepts clearly imply what the reader will soon discover for himself, that econometric model building is an art, and not a refined one at that. Economic data are often poor and ill-suited to testing hypotheses or estimating parameters. Economic theory varies enormously in its ability to provide guidance. Consumption theory provides clearcut testable hypotheses upon which we have put our reliance. Investment theory and wage theory, however, are in a confused state so we were more at liberty to experiment with moderately novel, although theoretically standard formulations of our own. Since our intent throughout has been to build a model that will have maximum theoretical acceptability subject only to a need for relative simplicity, departure from received theory and standard specification has been limited to sectors where the doctrine itself is not widely accepted. To the extent that we have successfully adhered to these principles, the econometric model can be viewed as a status report about the condition of macroeconometric theory and its econometric implementation.

As mentioned above, all behavioral equations in this model were estimated by ordinary least squares. Data from

the period 1954I - 1969IV were employed, (Roman numerals designate quarters, so that 1954I refers to the first quarter of 1954.) When the equations to be estimated are part of a system of simultaneous equations, as in this model, least squares estimates are known to be biased and inconsistent. We have used this method, rather than alternative approaches which eliminate bias in large samples, for two reasons. First, least squares is simpler and easier to interpret than the alternatives. This seems to us to be quite important in a textbook.³ Second, least squares estimates of macroeconomic stochastic equations seldom differ much from estimates produced by consistent estimation methods. }

Still another statistical problem should be mentioned. When lagged values of the dependent variable, values determined in earlier periods, are used as independent variables in a regression equation, and when the random error or disturbance terms in different periods are correlated, coefficient estimates are biased. Methods have been devised to correct for this, but we have not employed them. When disturbance terms in adjacent periods are correlated, the specification of the equation is often wrong. Something systematic is happening that is not being captured. Rather than make statistical assumptions about the behavior of what has been omitted, it seemed more sensible to us to

acknowledge that the equation is weak. Stated in more technical fashion, we normally prefer to interpret serial correlation of residuals as an indication of mis-specification, not as a sign that nature has generated serially dependent disturbance terms.

This model is not a set of linear equations; some equations are linear in the variables involved, but many are not. In fact, it would require fundamental changes in the structure of this model to make it linear. To cite only one example, the demand for Gross National Product is computed in current dollars by multiplying the constant dollar demand by the implicit GNP price deflator which, of course, produces a non-linear identity.

Several years ago a medium scale non-linear model would have been a severe computational burden. It would have been difficult to solve the model for the values of the endogenous variables. Now, thanks to modern computer hardware and software, this is no longer true. The TROLL system at MIT, on which this model was constructed, is able to solve large systems of non-linear equations with breathtaking ease.⁴ Non-linear models, however, do raise problems of interpretation. In particular, multipliers are not constant; they depend on the values of all variables. While this is unfortunate, neither we nor the practicing econometrics profession really believe that the economy can be adequately represented by a linear model.

It will be useful in our discussion of the various sectors or groups of equations which make up our model to refer to the following small Keynesian short-period macro-economic model, which resembles those found in most intermediate-level theory texts:

	<u>Sector</u>	<u>Chapter</u>
	(a) $C = C(YD)$	(3)
	(b) $I = I(r, Y)$	(4, 5)
	(c) $IM = IM(Y, P)$	(6)
	(d) $YD = YD(Y, t)$	(7)
	(e) $N = N(Y)$	(8)
(1.4)	(f) $W/P = f(Y, N)$	(9)
	(g) $MS/P = MD(Y, r)$	(10)
	(h) $Y = C + I + G + X - IM$	(12)

C = Consumption Expenditure

I = Investment Expenditure

IM = Imports

YD = Disposable Personal Income

N = Employment

W = Money Wage

P = Price Level

MS = Money Supply

MD = Money Demand

Y = Gross National Product

r = Rate of Interest

t = Tax Rates

G = Government Expenditure

X = Exports

The notation $C = C(YD)$ means that C is a function of YD , $I = I(r, Y)$ means that I is a function of r and Y and so on.

All Keynesian and post-Keynesian macroeconomic models follow (1.4) and begin with the determination of consumption spending (C), and ours is no exception. In Chapter 3 we examine the relation of consumption spending to disposable income, paying special attention to the lags involved in household behavior.

Equation (1.4.b) has a number of weaknesses as a basis for econometric investigations of investment (I). First, it ignores the levels of the stocks of housing, plant, equipment, and inventories in the economy. Second inventory investment spending behaves differently than investment spending for fixed assets. In Chapter 4 we examine the latter categories, while Chapter 5 deals with inventory investment.

Our short Chapter 6 contains an equation much like (1.4.d), which explains imports (IM) as a function of the level of economic activity and the price level. Exports (X), which are heavily dependent on foreign economic

conditions, are taken as exogenous to our model.

The relation of disposable personal income (YD) to GNP is not as simple as equation (1.4.d) suggests. Besides the impact of taxes, government transfer payments and corporate saving must be analyzed, along with depreciation allowances and a number of other quantities. Consequently, Chapter 7, which presents the equations relating disposable income to GNP, is rather long and involved.

Our examination of the level of employment (N) in Chapter 8 follows (1.4.e) and is concerned with the impact of the level of real output. Further, in order to forecast the unemployment rate, we investigate determinants of the labor force, i.e., the number of persons employed or seeking employment.

Most Keynesian textbook analysis takes the money wage (W) as fixed. Then an equation like (1.4.f) is presented and viewed as determining the price level as a function of income, employment, and the money wage. In Chapter 9 we follow this pattern and explain the price level as a function of unit labor costs. In addition, though, we treat money wages as a function of labor productivity and present equations for a number of particular prices appearing in the model.

In textbook models like (1.4), the nominal money supply (MS) is usually taken as exogenous and the rate of

interest (r) determined by the relation of the real money supply (MS/P) to the level of income. This simple formulation cannot be used directly, however. The money supply is not exogenous; it is determined by bank behavior, which the Federal Reserve can influence but not prescribe. Further, there are a multitude of interest rates in developed economies, each corresponding to a different financial asset. For these reasons, Chapter 10, which is devoted to financial markets and monetary policy, is fairly lengthy.

Government purchases from the private sector of the economy are taken to be exogenous, but the remainder of government expenditures (G), compensation of government employees, is determined by the level of government employment and private sector wage rates. The stochastic equation involved is presented in Chapter 11, along with a discussion of the National Income Accounts picture of government activity.

Finally, Chapter 12 presents some national income accounting identities (like (1.4.h)) that complete the model. Some of the properties of our model are also examined there. Appendix A on variables and Appendix B on the estimated equations should be useful as references to Chapter 12.

Thus each of Chapters 3-11 deals with a sector of the economy and with its representation in this model. The lines between sectors were often arbitrary, and we drew them where it seemed most sensible. Chapter 12 emphasizes the relations that tie the sectors together as well as the overall behavior of the model.

1.3 SECTOR PRESENTATIONS

In our discussion of each sector in Chapters 3-11, we first present and define the variables employed and then list the relevant identities. Next we discuss the estimates of the stochastic equations, mentioning approaches that failed as well as presenting our best estimates. The rationale for each equation in the model is spelled out as explicitly as seems necessary. In many cases, where standard, "off-the-shelf" specifications are employed, our specification discussion is not extensive and the interested reader should consult the references at the end of the chapter for more detailed treatments. We try throughout to indicate the strengths and weaknesses of the individual equation estimates.

Each sector is further tested and examined by means of two dynamic simulations. We now indicate what these are and how their output is to be interpreted.⁵ Consider the following modification of model (1.3):

$$(1.5) \quad \begin{aligned} C &= 20 + .5Y + .1Y(t-1) \\ Y &= C + I + G \end{aligned}$$

Now consumption depends on last period's income, $Y(t-1)$, as well as on current income, Y . Hereafter, lagged values are denoted by time subscripts, while the current period subscript is generally omitted. Given the values of the predetermined variables I , G , and $Y(t-1)$, model (1.5) can easily be solved for current period values of C and Y .

A dynamic simulation of this model would proceed as follows. Take the actual value of Y in some initial period zero and the values of I and G in period one and solve the model for estimates of C and Y in period one. (If, for instance, these values are 100, 30, and 30, respectively, the estimates for period one would be $Y = 180$, $C = 120$.) Then take the actual values of I and G in period two and the estimated value of Y in period one and solve for estimates of C and Y in period two. (If I and G were both 40 in period two, the estimates would be $Y = 236$, $C = 156$.) The simulation proceeds in this fashion for as many periods as desired. The essence of a dynamic simulation, then, is that the model generates its own current values of lagged endogenous variables and a complete set of exogenous variables.

The individual sectors and the entire model are simulated over the period 1954I-1969IV, the period to which

the behavioral equations were fitted. The estimated values of the endogenous variables are compared to the actual values. This simulation accomplishes two things. First, it gives a better idea than just regression statistics of the quality of the various equations. Second, it indicates whether any equations have a tendency to go off the track, generating estimates that are consistently too high or too low.

Tables in each chapter present the average value of each of the principal endogenous variables over this period. This information is useful in its own right to give one a sense of the relative importance of the variables. Second, we present the root-mean-squared (RMS) error for each endogenous variable, a basic tool for analyzing the accuracy of prediction. The error in each period is defined as the value computed by the simulation minus the actual value of the variable, so a negative error is an under-estimate. The root-mean-squared percentage error is also exhibited to allow for differences in scale among variables.⁶ The RMS absolute and relative errors give an intuitively useful idea of the quality of the estimate.

To see if variables are systematically over- or under-predicted, the arithmetic average of the errors is presented. In interpreting this figure, remember that a positive quantity means that the model over-estimated the

variable in question on average. We also present the results of a simple t-test of the hypothesis that this mean error equals zero. This is by no means an exact test for data generated in this manner, but rejection of the hypothesis that the mean error is zero provides some indication that the variable has strayed systematically.

Besides the 1954-1969 simulations, we have simulated each sector [and the model] as a whole over the four quarters of 1970. The statistics just discussed are presented for these simulations also. The purpose of these runs is to see how well the various sectors perform outside the sample period. Large RMS errors or large t-statistics generally indicate specification problems.

Model builders, including ourselves, are inclined to "mine the data", i.e., to estimate a variety of different equation forms until one is found with acceptable error variance (i.e., small), coefficient signs and t-statistics. Tests made outside the period of fit provide some insurance against this abuse of statistical theory (however compelling the procedure appears to be), although four observations by no means provide air-tight assurance.

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Footnotes for Chapter 1

1. Nerlove [1966] provides a useful survey of macro-econometric models. A number of different types of models are described in the works listed in the second set of references at the end of this chapter.
2. For other discussions of econometric modeling, see Christ [1966, Part I], and Liebenberg, et al. [1966, pp. 13-16].
3. Furthermore, there is no real concensus within the profession as to the best simultaneous equations estimator. Even if the popular instrumental variables approach is chosen, many arbitrary decisions must go into selection of the instrumental variables to employ for each equation in small samples. Hence readers interested in reproducing and then modifying our calculations are best provided, as we have done, with least square estimates.
4. It takes less than three seconds to solve the model for one period on the TROLL system as implemented on the IBM 360/67 system. See Eisner [1969] for a description of an earlier version of the system.
5. See also Fromm and Taubman [1968].
6. When the variable can assume negative as well as positive values, this statistic is no longer very useful, though it is difficult to conceive of a replacement.

CHAPTER 2

DISTRIBUTED LAGS

2.1 INTRODUCTION

Much economic theory is static; time does not enter in an essential way. Static theory, though, seldom provides all the information needed to model the real world. Suppose, for instance, that the static theory implies that some variable Y depends on another variable, X . If the quantity Y represents an outcome of individuals' decision processes, such as total consumption or total investment, it is unlikely that changes in X will be immediately reflected in Y . It is often quite important, especially for policy decisions, to be able to characterize the lag involved, to determine how long it takes Y to respond to changes in X .¹

Individual decision-makers are likely to respond to changes in X some time after they occur, but not all people will wait the same length of time to act. If they did, changes in Y would lag changes in X by some fixed length of time. If individuals' lags differ, the aggregate response to changes in X will be spread over more than one period of time. Such lags are called distributed lags.² These may exist at the individual level as well as in the aggregate if individuals consider more than one

lagged value of X in making decisions. Distributed lags at the household level are suggested by the Permanent Income hypothesis of consumer spending behavior, for instance, as we discuss in the next chapter.

If Y is determined by X through a distributed lag, this means that Y depends on more than one past value of X . If we assume a linear model, we can write the general distributed lag relation as

$$(2.1) \quad Y(t) = a[w_0 X(t) + w_1 X(t-1) + w_2 X(t-2) + \dots].$$

? The w 's add to one.³ It is sometimes assumed that they are all positive, although sensible distributed lags may arise where some weights are negative. There may or may not be an infinite number of w 's. (The difference between a rapidly diminishing infinite series and a finite series is difficult to detect in practice, and the former is often simpler to manipulate algebraically.) The constant a gives the eventual change in Y that occurs in response to a maintained unit change in X .

In what follows, we examine two aspects of distributed lags. The first is the relations among alternative lag specifications. Our second and major concern is with techniques for obtaining analytical insight into the dynamic properties of an already estimated dynamic relationship.

In the real world we must deal with, there is never enough high quality economic data to permit direct statistical estimation of a large and perhaps infinite number of w 's. Direct estimation of equations like (2.1) is just not generally feasible. The usual approach is to make some assumptions about the shape of the sequence of w 's. The sequence is described by a small number of parameters, and these parameters are then estimated from the data. Once a particular approximation of this sort is decided upon, theoretical analysis can proceed in terms of the parameters of the sequence of w 's, and analysis of the estimated lag distribution can be conducted similarly.

For example, a recent approach is to assume that the sequence of w 's is finite and can be adequately approximated by a polynomial function of the lag involved. If one then specifies the degree of the polynomial, its coefficients can be estimated. For instance, one might assume that

$$(2.2) \quad w_i = A + Bi + Ci^2 \quad (i=0,1,\dots,8),$$

where A , B , and C are constants to be estimated, and $w_i = 0$ for i greater than or equal to nine. Using a computational method proposed by S. Almon [1965], the three unknown parameters can be estimated statistically.⁴ By placing reasonable prior restrictions on the lag distribution in this

fashion, one can greatly reduce the number of coefficients that must be estimated (from nine to three in our example, since α must be estimated also), while not unduly restricting the shape of the distribution.

This approach has the drawback that one must specify in advance the number of non-zero w 's and the degree of the polynomial. One ends up conducting an extensive search for the "best" combination, without any clear-cut theoretical guidance. Also, if seven lagged X 's are assumed to influence Y , one must begin estimation with the eighth observation on Y . With long lags and few available observations, a good deal of information may be lost this way, although the same problem arises to some extent in all lag estimation methods. These Almon or polynomial lags may be useful when the lag is known to be finite, and where many observations are available. We employ this technique in such situations in the Appendix to Chapter 4 and in Chapter 5.

The second approach, more common throughout this model and in other econometric work, is to assume that there are an infinite number of non-zero w 's. For this to make any sense, w_i has to fall rapidly to zero as i , the lag index, becomes large. The simplest assumption of this sort is

$$(2.3) \quad w_i = (1-k)k^i,$$

Here k must be a constant between zero and plus one in order for the sum of the w_i to equal one, since k^i is a geometric progression which sums to $1/(1-k)$. Notice that the w_i are all positive and that the distributed lag is completely described by two parameters, k and a . The assumptions about the w_i expressed by equation (2.3) were first proposed and explored by Koyck [1954], and we speak of this as a Koyck or geometric (or first-order) distributed lag.

Under the Koyck assumption, a one period increase in X followed by a return to its previous level causes an increase in Y followed by a gradual decrease to its earlier level. (See Section 2.2 for a proof.) Another common pattern of response involves a small initial change in Y which gradually builds to a peak and then falls to small values once more. Simple two or three parameter variants of the Koyck lag exist that can readily describe the corresponding patterns of lag weights; see Solow [1960], Jorgenson [1966], and Section 2.3 below.

The beauty of the Koyck structure and its more complex variants is that equation (2.1) can be rewritten so as to involve $X(t)$ and a few lagged values of Y . In fact, if there are N parameters like k in (2.3) that determine the lag structure, (2.1) can be rewritten to involve just N lagged values of Y .

In the next section, we examine the properties of the Koyck lag in detail. While this lag distribution is basically quite simple, it is a natural vehicle to employ to explain techniques which are useful in analyzing more complex structures. Section 2.3 carries out such an analysis.

2.2 THE KOYCK OR GEOMETRIC LAG

We first verify that assumption (2.3) permits us to drastically simplify equation (2.1). Substituting (2.3) into (2.1),

$$\begin{aligned}
 (2.4) \quad Y(t) &= a(1-k) \sum_{i=0}^{\infty} k^i X(t-i) \\
 &= a(1-k)X(t) + a(1-k) \sum_{i=1}^{\infty} k^i X(t-i).
 \end{aligned}$$

Notice that the smaller is k , the more rapidly the influence of past X 's decays. Lagging (2.4) by one period and multiplying by k , we have

$$\begin{aligned}
 (2.5) \quad kY(t-1) &= a(1-k)k \sum_{i=0}^{\infty} k^i X(t-1-i) \\
 &= a(1-k) \sum_{i=1}^{\infty} k^i X(t-i)
 \end{aligned}$$

Subtracting (2.5) from (2.4), we obtain

$$(2.6) \quad Y(t) = a(1-k)X(t) + k Y(t-1).$$

Two properties of this equation are immediately obvious. If $X(t)$ is increased by one unit, $Y(t)$ will rise by $a(1-k) = a w_0$ units. If X is increased by one unit and is maintained at the new level forever, $Y(t)$ must eventually equal $Y(t-1)$, provided k is less than one in absolute value. Setting $Y(t)$ equal to $Y(t-1)$ and solving, we obtain $Y(t) = aX(t)$. Both these properties agree with equation (2.1). Further, we shall verify below that if k is between zero and one, a maintained change in X will cause Y to steadily (monotonically) approach its new equilibrium. The larger is k , the more important is past history, and the slower Y approaches equilibrium.

We shall now examine parameters used to summarize lag distributions, and we shall evaluate these quantities for the Koyck lag structure. Clearly this structure is easily if not insightfully summarized by the parameter k , but the summary parameters we shall consider and (especially) the way we shall find them will be useful in the consideration of more complex lag structures.

Adjustment Time and Median Lag

In equation (2.6), suppose that $Y(0) = aX(0)$. That is, assume that the system is in equilibrium in period zero. Suppose $X(1) = X(0) + 1$, and that this value of X is maintained thereafter. Then $Y(1) = a(1-k) [X(0) + 1] + kY(0) = Y(0) + a(1-k)$.

Substituting further, we find

$$\begin{aligned} Y(2) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)] \\ &= Y(0) + a(1-k)(1+k), \\ Y(3) &= (1-k)Y(0) + a(1-k) + k[Y(0) + a(1-k)(1+k)] \\ &= Y(0) + a(1-k)(1+k+k^2), \end{aligned}$$

and in general, summing the geometric series in k ,

$$(2.7) \quad Y(t) = Y(0) + a(1-k^t). \quad (t = 0, 1, \dots)$$

The new equilibrium value of Y will be $Y_e = Y(0) + a$, as asserted above. Notice that if k is between zero and one, the difference between $Y(t)$ and Y_e declines steadily over time, also as asserted above.

The above analysis describes what engineers would call the step response of the system. We can also calculate the impulse response, that is, the response of Y to a change in X which is not maintained. Let the system be in equilibrium in period zero as before. Now assume that $X(1) = X(0) + 1$, but $X(t) = X(0)$ for t greater than or equal to two. Substituting,

$$\begin{aligned} Y(1) &= a(1-k)[X(0)+1] + k[aX(0)] \\ &= aX(0) + a(1-k) \\ Y(2) &= a(1-k)[X(0)] + k[aX(0)+a(1-k)] \\ &= Y(0) + a(1-k)k \\ Y(3) &= a(1-k)(X(0)) + k(Y(0)+a(1-k)k) \\ &= Y(0) + a(1-k)k^2. \end{aligned}$$

In general, we clearly have

$$(2.8) \quad Y(t) = Y(0) + a(1-k)k^{t-1} \quad t=1,2,\dots)$$

The transient increase in X causes Y to rise by $a(1-k)$ in the first period and then to fall gradually to equilibrium, as asserted in Section 2.1, above.

Considering (2.7), the fraction of the adjustment to the new equilibrium completed in the first t periods after a maintained change in X begins is simply

$$(2.9) \quad \frac{Y(t) - Y(0)}{Y_e - Y(0)} = \frac{a(1-k^t)}{a} = (1-k^t).$$

The median lag, T_{md} , is simply that value of t for which the fraction of adjustment completed equals one half.

Thus we have

$$(2.10) \quad .5 = 1 - k^{T_{md}}, \text{ or } T_{md} = \log (.5) / \log (k).$$

Note that as k goes to zero, so does the median lag, as one might expect. The median lag is less than or greater than one period, depending on whether k is less than or greater than .5.

The fraction of readjustment completed after t periods- and from this, the median lag-can also be derived from consideration of the equation's impulse response, since from (2.8) the fraction of the distance that Y was displaced from equilibrium that has been made up t periods after X has returned to $X(0)$ is

$$(2.11) \quad \frac{Y(1) - Y(t+1)}{Y(1) - Y(0)} = (1 - k^t),$$

which equals (2.9).

In complicated lag structures, the median lag may be hard to compute. In its place, we usually use the mean lag, T_m , to measure the speed of response. The mean lag is defined by

$$(2.12) \quad T_m = \sum_{i=0}^{\infty} i w_i.$$

Before computing the mean lag for the Koyck structure, it will be useful to introduce two tools of broad application.

First, though, we should mention a useful way of looking at mean and median lags. If all the w 's in some lag distribution are non-negative, they may be thought of as probabilities, since they add to one. Then the mean lag is just the mean of the corresponding random variable which equals i with probability w_i , and the median lag is one plus its median. Clearly if some of the w_i are negative this interpretation is not useful - but then neither are the mean and median lags, in general.

Operators and Lag Polynomials

We now introduce two operators which are employed throughout this book. The first is the lag operator, which we write as L . This operator is defined by the following identity, where Z is any time-series variable:

$$(2.13) \quad L^k Z(t) = Z(t-k).$$

The other important operator is the difference operator, Δ , which is defined in terms of L by

$$(2.14) \quad \Delta = 1-L, \text{ or } L = 1- \Delta.$$

Thus for any time-series variable Z ,

$$\Delta Z(t) = (1-L)Z(t) = Z(t) - Z(t-1),$$

$$\begin{aligned} \Delta^2 Z(t) &= (1-L)^2 Z(t) = (1-2L+L^2)Z(t) \\ &= Z(t) - 2Z(t-1) + Z(t-2), \end{aligned}$$

and so on.

Writing the general distributed lag equation (2.1) in terms of the lag operator, we obtain

$$(2.15) \quad Y(t) = \left[\sum_{i=0}^{\infty} w_i L^i \right] aX(t) = P(L) aX(t).$$

The quantity in brackets in (2.15), denoted $P(L)$, is called a lag polynomial, which is just shorthand for "polynomial in the lag operator with coefficients that sum to one." Equation (2.15) has a number of advantages over the equivalent representation (2.1), as we shall see.

Mean Lag

Given a distributed lag equation in the form of (2.15), it is a simple matter to obtain the mean lag. First differentiate $P(L)$ term by term with respect to L , and then replace L by 1. Examination of (2.12) and (2.15) should convince you that $T_m = P'(1)$.

From (2.3), equation (2.15) in the Koyck case becomes

$$\begin{aligned}
 (2.16) \quad Y(t) &= \left[(1-k) \sum_{i=0}^{\infty} k^i L^i \right] a X(t) \\
 &= \left[\frac{1-k}{1-kL} \right] a X(t).
 \end{aligned}$$

The second line was obtained from the first by treating L like an ordinary constant between zero and one, and expressing the sum of the geometric series $(kL)^i$ in closed form. Notice that if we multiply both sides of (2.16) by $(1-kL)$ and substitute $Y(t-1)$ for $L Y(t)$, we obtain equation (2.6). Differentiating the lag polynomial in the second line of (2.16) with respect to L and setting L equal to one, we obtain:

$$(2.17) \quad T_m = P'(1) = \frac{k}{1-k}$$

as with the median lag, when k goes to zero the mean lag does also.

Variance of Lag Distribution

The variance of the lag distribution, V_L is another useful magnitude for lag distributions with positive w_i . It expresses how much the influence of X is spread out over time, a dimension not measured by the mean or median lag. This quantity is defined by

$$(2.18) \quad V_L = \sum_{i=0}^{\infty} w_i \left[i - T_m \right]^2 = \sum_{i=0}^{\infty} i^2 w_i - T_m^2.$$

An examination of (2.15) should make it clear that the first term in (2.18) is found by differentiating the lag

polynomial twice with respect to L , setting L equal to one, and adding the mean lag. In the Koyck case,

$$\begin{aligned}
 (2.19) \quad V_L &= P''(1) + P'(1) - [P'(1)]^2 \\
 &= \frac{2k^2}{(1-k)^2} + \frac{k}{(1-k)} - \frac{k^2}{(1-k)^2} = \frac{k}{(1-k)^2}
 \end{aligned}$$

Individual Lag Weights

The lag polynomial has one other important use. From it, we can readily compute the lag weights. These would otherwise be difficult to obtain in complicated lag structures. Notice that the N^{th} derivative of the lag polynomial in (2.15) with respect to L is given by

$$(2.20) \quad \frac{d^N P(L)}{dL^N} = \sum_{i=N}^{\infty} \frac{i!}{(i-N)!} w_i L^{i-N},$$

since the terms corresponding to i less than N vanish identically. (Recall that $N! = N(N-1)(N-2)\dots(2)(1)$).

Setting $L=0$, all terms with i greater than N vanish.⁶ Since $0!$ is identically equal to one, we have the result

$$(2.21) \quad \left. \frac{d^N P(L)}{dL^N} \right|_{L=0} = N! w_N.$$

We can thus go back uniquely from the lag polynomial to the w 's. This relation is easy to verify in the Koyck case using (2.16), and it is occasionally useful in higher-

order structures.

Before examining such structures, it will be useful to illustrate the application of the tools we have developed. Suppose we have estimated a distributed lag relation between Y and X and have obtained the following equation:

$$(2.22) \quad Y(t) = .30 X(t) + .80 Y(t-1).$$

The short-run impact of X upon Y is simply .30. To obtain more information, compare equation (2.22) to equation (2.6). It is clear that $k = .80$ and that a , the long-run impact of X on Y , is equal to $.30/(1-.80) = 1.50$. Using equation (2.10), we can compute the median lag:

$$T_{md} = (\log(.5)/\log(.8)) = (-.693)/(-.223) = 3.11 \text{ periods.}$$

From equation (2.17), the mean lag is simply $.80/(1-.80) = 4.00$ periods. Similarly, equation (2.19) could be used to compute the variance of the lag distribution, and equation (2.21) could be employed to compute the individual lag weights.

Now suppose that the estimated relation between X and Y had been

$$(2.23) \quad Y(t) = .20 X(t) + 1.10 Y(t-1).$$

Can we compute similar statistics for this equation? No, since the implied value of k , 1.10, is not consistent with the Koyck lag scheme. It may be possible to make sense of equation (2.19), but it cannot be interpreted as an estimate

of a geometric distributed lag function.⁷

2.3 MORE GENERAL LAG MECHANISMS

We shall begin this brief discussion of more complicated lag schemes with a second-order example. Suppose that the quantity X in (2.16) represents an observed data series, but that Y is not observable. For instance, in an investment study, X might be sales and Y might be decisions to purchase new capital goods. No data on decisions to purchase new capital are readily available, but it is desired to explain investment spending, Z . We assume that Z is observable and that it is related to Y according to

$$(2.24) \quad Z(t) = \left[\frac{1-m}{1-mL} \right] b Y(t),$$

where L is the lag operator, as before, and m is a constant between zero and one. The mean lag of (2.24) is clearly $m/(1-m)$. In the investment study, this would represent the mean lag between decisions and deliveries. We can combine (2.16) and (2.24) in a way that expresses $Z(t)$ as a function only of the observable variable $X(t)$:

$$(2.25) \quad Z(t) = \left[\frac{(1-m)(1-k)}{(1-mL)(1-kL)} \right] a b X(t).$$

By differentiating the lag polynomial in brackets in (2.25) with respect to L and setting L equal to one, it can be shown that the mean lag in (2.25) is equal to

$[m/(1-m)] + [k/(1-k)]$. The mean lags thus add when linear equations are combined in this fashion. The variance of the lag can also be computed, and equation (2.21) could be used to compute the lag weights, the w_i .

Multiplying (2.25) though by $(1-mL)(1-kL)$ and rewriting we obtain⁸

$$(2.26) \quad Z(t) = [(1-m)(1-k)ab] X(t) + (k+m) Z(t-1) - km Z(t-2).$$

Suppose we estimate the coefficients of (2.26) from time-series data on $Z(t)$ and $X(t)$. The question may arise whether the estimated coefficients can be interpreted as having come from a dynamic structure that has a monotonic approach to equilibrium. Let our estimated equation be

$$(2.27) \quad Z(t) = A X(t) + B Z(t-1) + C Z(t-2).$$

It can be shown that this function will imply a set of w_i greater than zero, and hence a monotonic approach to equilibrium, if the following conditions are satisfied:⁹

$$(2.28) \quad \begin{aligned} 2 > B > 0, & \quad -1 < C < 1, \\ B + C < 1, & \quad B^2 \geq -4C. \end{aligned}$$

In the general case, distributed lag equations may involve many lagged Z 's, and there may be lagged X 's as well.

The restrictions analogous to (2.28) that estimated coefficients must satisfy in order to represent sensible lag structures will then be quite complex; they will not concern us here.¹⁰

To examine the general case, we define:

$$(2.29) \quad \begin{aligned} F(L) &= a_0 + a_1 L + a_2 L^2 + \dots + a_m L^m \\ G(L) &= 1 - b_1 L - b_2 L^2 - \dots - b_n L^n \end{aligned}$$

The general difference equation may then be rewritten as a rational distributed lag:

$$(2.30) \quad Z(t) = \left[\frac{F(L)}{G(L)} \right] X(t)$$

This is called a rational lag since the lag polynomial $P(L)$ may be written as the ratio of two polynomials in L .¹¹ Notice that the long-run impact of $X(t)$ on $Z(t)$ is given by $F(1)/G(1)$; this is the change in equilibrium Z brought about by a maintained unit change in X .

We can rewrite (2.30) in the same form as (2.25), by dividing $F(L)$ by the scalar $F(1)$ and $G(L)$ by the scalar $G(1)$, so that the coefficients of both lag polynomials normalized in this fashion add to unity; to retain equation (2.30) as written originally, the expression in brackets must be multiplied by $F(1)/G(1)$:

$$(2.31) \quad Z(t) = \left[\frac{F(L)/F(1)}{G(L)/G(1)} \right] \frac{F(1)}{G(1)} X(t) = \left[\frac{G(1)}{F(1)} \frac{F(L)}{G(L)} \right] \frac{F(1)}{G(1)} X(t)$$

The quantity in brackets is a lag polynomial; the original coefficients (the aw_i in (2.1)) have been multiplied by the scalar quantity $[G(1)/F(1)]$ ($1/a$ in (2.1)) so that the sum of the lag weights is unity. Once an equation has

been written in this form, with the lag polynomial factored out, that polynomial can be differentiated with respect to L to find the mean lag, the variance of the lag, and the individual lag weights. Note that the procedures we derived earlier for computing these magnitudes cannot be used directly on $[F(L)/G(L)]$; an important assumption in those earlier derivations was that the sum of the coefficients of the lag polynomial equalled unity.

We conclude this chapter with an illustration of the use of the tools developed here. Consider the following estimated equation.

$$(2.32) \quad Y(t) = 1.0 X(t) + 2.0 X(t-1) + 1.10 Y(t-1) - .20 Y(t-2).$$

Conditions (2.28) are satisfied, so the w_i are all positive. The initial impact of X on Y is simply 1.0, while the long-run effect of a maintained unit change in X is given by $(1.0 + 2.0)/(1.0 - 1.10 + .20) = 30$.

To obtain further results, we rewrite (2.32) in the form of (2.31). Here we have

$$F(L) = 1.0 + 2.0 L; F(1) = 3.0$$

$$G(L) = 1.0 - 1.10 L + .20 L^2; G(1) = .10$$

Hence (2.32) may be written as

$$(2.33) \quad Y(t) = 30. X(t) \left[\frac{(.10)(1 + 2L)}{(3)(1 - 1.1L + 2L^2)} \right].$$

Differentiating the lag polynomial in brackets with respect to L and evaluating the derivative at $L = 1$, we obtain the

mean lag:

$$T_m = \frac{.10}{3.0} \frac{.2 + 2.1}{.01} = 7.7 \text{ periods.}$$

We could compute the variance of the lag distribution similarly, and equation (2.21) could be used to obtain the lag weights. There is an easier way to obtain the initial lag weights in this case, however, and we now illustrate it.

The lag polynomial can be written in the following form after some trivial rearrangement:

$$(2.34) \quad \left[\frac{(1 + 2L)}{30} \right] \left[\frac{1}{[1 - (1.1L - .2L^2)]} \right]$$

The second term is simply the sum of a geometric series.¹²

Writing the series out, we obtain

$$\begin{aligned} & \frac{(1 + 2L)}{30} \cdot \sum_{i=0}^{\infty} (1.1L - .2L^2)^i \\ &= \frac{1 + 2L}{30} [1 + 1.1L - .2L^2 + 1.21L^2 - .22L^3 + .04L^4 + \dots] \\ &= (1/30) [1 + 3.1L + 3.21L^2 + \dots] \end{aligned}$$

Thus we have $w_0 = (1/30)$, $w_1 = (3.1/30)$, and $w_2 = (3.21/30)$.

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Footnotes for Chapter 2

1. Much of the current controversy about the relative merits of monetary and fiscal policies centers on the length and stability of lags in the effect of changes in the quantity of money or in taxes and government spending on the aggregate variables they presumably influence.
2. Griliches [1967] provides a useful survey of much of the material covered in this chapter as well as a discussion of the statistical problems associated with distributed lag specifications.
3. The w_i can be chosen so that their sum is unity simply by dividing each original lag coefficient by the sum of these coefficients; that sum then becomes the constant a in (2.1). The significance of this normalization will become clear later in the chapter.
4. See Tinsley [1967] for a generalization of this approach.
5. The reader should set up a lag polynomial, follow the rules given, and see that the result follows immediately. This form of analysis has been taken directly from the generating functions (or transforms) used in probability theory: see Drake [1967, Ch. 3] and Feller [1968, Ch. 11].

6. If the reader will work out an example with a third degree lag polynomial for $N=3$, the result shown in (2.21) will become immediately evident.
7. Of course, if the standard errors of the estimated coefficients are large enough, it may be impossible to refute the hypothesis that the coefficient of the lagged dependent variable is less than one and thus that the true structure embodies a Koyck lag. But this is cold comfort, since it will still be hard to use (2.23) sensibly or to place much confidence in it.
8. It can be shown that for k and m less than one, all the w_1 will be positive. Thus changes in X will cause Z steadily to approach its new equilibrium.
9. Conditions (2.28) originate from restrictions on the roots of the second-order difference equation in $Z(t)$, equation (2.27). These roots must be real, positive, and less than unity. See Baumol [1970] and Goldberg [1958] for general treatments of difference equations.
10. See Jorgenson [1966].
11. Equation (2.30) makes no sense as a representation of a distributed lag mechanism unless $G(1)$ is positive, because otherwise a stable equilibrium does not exist. Jorgenson [1966], analyzes the properties of such structures.

12. Here as before we use the identity

$$\sum_{i=0}^{\infty} Z^i = \frac{1}{1-Z} \quad (-1 < Z < +1).$$

In this instance, $Z = 1.1L - .2L^2$.

CHAPTER 3

PERSONAL CONSUMPTION EXPENDITURE

In this chapter we present and discuss our consumption sector, which determines total consumption expenditures in constant dollars, as a distributed lag function of disposable personal income and the implicit price deflator for consumption. We first describe the data series employed in this sector and then examine two widely-used dynamic models of consumer spending behavior. The equations of this sector are then presented; two approaches to the prediction of total consumption spending are discussed. Dynamic simulation results of these two approaches to consumption behavior conclude the chapter.

3.1 THE DATA

The data series appearing in this chapter and in the consumption sector are listed in Table 3.1. The population series, LTPOP, is the quarterly average of monthly figures taken from the Survey of Current Business. The three aggregate series CTOT, YDPI, and PCTOT were taken directly from the National Income Accounts.¹ The first three per capita quantities shown, CPTOT, CPND, and CPDUR were computed from total constant dollar consumption in these categories (again from the National Income Accounts) and LTPOP.

Table 3.1

Variables Appearing in Chapter 3
and in the Consumption Sector

(Endogenous Variables - Discussed in Chapter 3)

CTOT	Total Consumption Expenditures (billions of constant dollars, SAAR).
CPTOT	Total Consumption Expenditures Per Capita (constant dollars, SAAR).
CPND	Per-Capita Consumption Expenditures on Non-Durables (constant dollars, SAAR).
CPDUR	Per-Capita Consumption Expenditures on Consumer Durables (constant dollars, SAAR).
CPHS	Per-Capita Consumption Expenditures on Housing Services (constant dollars, SAAR).
CPNS	Per-Capita Consumption Expenditures on Services Other Than Housing Services (constant dollars, SAAR).
Y	Per-Capita Real Disposable Income (constant dollars, SAAR).

(Endogenous - Determined Elsewhere in the Model)

YDPI	Disposable Personal Income (billions of current dollars, SAAR).
PCTOT	Implicit Price Deflator for Total Consumption Expenditures (1957-59 = 100).

(Exogenous Variable)

LTPOP	Total Population (billions of persons)
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SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual Rates

Implicit deflators for the various categories of services consumption are available in the Accounts only on an annual basis, while current dollar spending by category and the deflator for total services are published quarterly. The annual deflator for housing services is nearly a pure trend, however, so we confidently interpolated to obtain a quarterly series. This permitted us to calculate housing services consumption in constant dollars and, by subtraction, non-housing services in constant dollars. Then LTPOP was used to obtain CPHS and CPNS.

All flow variables based on the National Income Accounts (all series mentioned in Table 3.1 except LTPOP and PCTOT) are seasonally adjusted quarterly totals measured at annual rates.

3.2 BASIC SPECIFICATIONS TESTED

A number of issues had to be resolved before hypothetical consumption equations could be written. The first question was whether or not to include relative prices. It seemed unlikely that there are substantial price effects as among broad categories of consumer expenditure. Some macro-models find statistically significant price terms, but they contribute little to explained variance. In addition, the consumption deflators are known to be subject to substantial measurement error, and they are inaccurately forecast

in a macro-model. We thus decided to exclude price terms from the consumption equations.

Most theory treats consumer demand in real terms, and we have accepted that formulation. Thus consumers are not assumed to suffer from "money illusion", though this assumption has recently been challenged by Branson and Klevorick [1969].

There has been a gradual shift of expenditure to services and away from non-durables measured in constant dollars, with the overall average propensity to consume holding steady at about .92. This suggests that the long-run average propensities to spend on the various categories have been shifting. Efforts to capture these shifts by introducing quadratic income terms were unsuccessful.

It would appear that the most sensible measures of consumption are in per capita terms. It should make some difference in aggregate consumption behavior whether a given amount of disposable income is divided among 100 million people or 200 million people, when positive saving exists and is responsive to household size. By working with per capita quantities, we add an element of reality and, as a bonus, remove some of the common trend from the variables.³

Once the decision to work with per capita variables in constant dollars had been made, most of the specification was complete.

Particular functional forms must still be selected, and there was some uncertainty as to the nature of the distributed lags involved. To examine the two basic models that underlay our exploration, we define the following symbols:

$Y(t)$ = aggregate real disposable income in period t

$C(t)$ = real consumption in period t in the category of interest

$P(t)$ = population in period t

$y(t) = Y(t)/P(t)$ = per capita current income

$y_p(t)$ = per capita permanent income

$c(t) = C(t)/P(t)$ = per capita consumption

$r(t)$ = per capita net stock, usually of durables, at the end of period t

The most common sort of consumption function found in the literature rests on the partial adjustment assumption. Ordinarily, the assumption is made that there exists a target level of per capita consumption given by

$$(3.1) \quad c^*(t) = \alpha + \beta y(t).$$

Actual consumption is adjusted towards c^* according to

$$(3.2) \quad c(t) - c(t-1) = \gamma[c^*(t) - c(t-1)], \quad \text{or} \quad \gamma c^*(t) + (1-\gamma)c(t-1) = c(t)$$

where γ is a constant between zero and one.

According to this formulation, consumption is smoothed and short-run fluctuations in income have their main impact upon savings. If we substitute the first equation into the second, we obtain the form actually estimated:

$$(3.3) \quad c(t) = \alpha\gamma + \beta\gamma y(t) + (1-\gamma)c(t-1).$$

Equation (3.3) can also be derived as a consequence of a simple version of Friedman's [1957] Permanent Income hypothesis. Suppose consumption is given by

$$(3.4) \quad c(t) = \alpha + \beta y_p(t),$$

where $y_p(t)$ is per capita permanent income in period t . This quantity is most often approximated by the following geometrically weighted sum of prior incomes:

$$(3.5) \quad y_p(t) = \gamma \sum_{i=0}^{\infty} (1-\gamma)^i y(t-i) \\ = \frac{\gamma}{1-(1-\gamma)L} y(t),$$

where L is the lag operator introduced and discussed in Chapter 2.

Substituting (3.5) into (3.4), we have

$$(3.6) \quad c(t) = \alpha + \frac{\beta\gamma}{1-(1-\gamma)L} y(t).$$

Multiplying through $[1-(1-\gamma)L]$ and substituting $c(t-1)$ for $Lc(t)$, we obtain

$$(3.7) \quad c(t) = \alpha\gamma + \beta\gamma y(t) + (1-\gamma)c(t-1),$$

which is identical to (3.3).

It is worthwhile rewriting equation (3.3) in terms of aggregate quantities. Let us denote the estimate of $\alpha\gamma$ by b_0 , the estimate of $\beta\gamma$ by b_1 , and the estimate of $(1-\gamma)$ by b_2 :

$$(3.8) \quad C(t) = b_0 P(t) + b_1 Y(t) + b_2 \frac{C(t-1)P(t)}{P(t-1)}$$

The short-run effects of income and population are given by:⁴

$$SRMPC = b_1 \quad SRMPE = b_0 + b_2 \frac{C(t-1)}{P(t-1)}.$$

(SRMPE is the short-run marginal population effect, the current quarter change in total consumption caused by adding one to the population.) If an increase in income or population is maintained long enough for consumption to stabilize, $C(t) = C(t-1)$ so that

$$\text{and } P(t) = P(t-1)$$

$$LRMPC = b_1/(1-b_2) = \beta, \quad LRMPE = b_0/(1-b_2) = \alpha.$$

The mean lag in (3.3) and (3.7) is clearly $(1-\gamma)/\gamma$, from the last chapter. This is the mean lag of $c(t)$ behind $y(t)$; there is no corresponding estimate for $C(t)$ and $Y(t)$, as equation (3.8) is

not a distributed lag relation in the usual sense.⁵

Note that population growth will almost always increase current aggregate consumption, but that it will have a positive long-run effect only if the estimated equation has a positive intercept.

The above derivation is sensible if $c(t)$ measures real purchases of non-durable goods or services, for then $c(t)$ can be thought of as directly entering the typical consumer's utility function and thus, via the usual microeconomic arguments, as being dependent upon income. The permanent income theory can then be viewed as extending the usual textbook discussion by taking explicit account of the dynamic nature of consumer decisions and uncertainty regarding future incomes. But if $c(t)$ represents purchases of durable goods, the justification for (3.4) is considerably weaker.

In this latter case, the usual assumption is that the services of durable goods, rather than purchases, directly enter the utility function.⁶ If we assume that the level of services provided the typical consumer at any time by the durable goods he owns is proportional to the net stock, $s(t)$, of such goods, a logical replacement for (3.4) is

$$(3.9) \quad s(t) = \alpha + \beta y_p(t).$$

If we assume that the net stock of durables is reduced by a constant fraction μ each period because of depreciation, the identity connection $c(t)$ and $s(t)$ is

$$(3.10) \quad \begin{aligned} c(t)/4 &= s(t) - (1-\mu)s(t-1) \\ &= [1-(1-\mu)L]s(t). \end{aligned}$$

The constant 4 is present in (3.10) because we now add the assumption (which applies in our model) that $c(t)$ and $y(t)$ are quarterly totals measured at annual rates. This implies that the actual spending in a given quarter and thus the gross addition to $s(t)$ in that quarter is equal to $c(t)/4$.

Combining (3.9) and (3.10) and making use of the lag operator, we have

$$(3.11) \quad c(t)/4 = [1-(1-\mu)L][\alpha + \beta y(t)].$$

If we assume that permanent income is determined by (3.5), as before, we can substitute into (3.11) and obtain

$$(3.12) \quad c(t)/4 = \alpha\mu + [1-(1-\mu)L] \frac{\beta\gamma}{1-(1-\gamma)L} y(t)$$

Multiplying through by $[1-(1-\gamma)L]$, substituting and collecting terms, we have

$$(3.13) \quad c(t) = \alpha\mu\gamma + \beta\gamma\Delta y(t) + \beta\gamma\mu y(t-1) + (1-\gamma)c(t-1),$$

where Δ is the difference operator introduced in Chapter 2.

Hendrik Houthakker and Lester Taylor [1966] present another derivation of a consumption equation of the form of (3.13). They begin with the assumption

$$(3.14) \quad c(t)/4 = \alpha + \beta y(t)/4 + \delta s(t-1).$$

Again we are assuming that $c(t)$ and $y(t)$ are quarterly flows measured at annual rates. Note that $s(t-1)$ is the stock on hand at the start of period t . In the case of durables, $s(t)$ may be interpreted as the real per capita stock of consumer durables on hand. Houthakker and Taylor extend this approach beyond durable goods, however. They interpret $s(t)$ as a psychic "stock of habits" in the case of non-durables and services. In either case, we still assume that identity (3.10) holds.

Multiplying both sides of (3.14) by $[1-(1-\mu)L]$ yields

$$(3.15) \quad [1-(1-\mu)L]c(t)/4 = \alpha\mu + \beta[1-(1-\mu)L]y(t)/4 \\ + \delta[1-(1-\mu)L]s(t).$$

From (3.10), the last term is equal to $\delta c(t-1)/4$. Substituting and re-arranging terms, we obtain

$$(3.16) \quad c(t) = 4\alpha\mu + \beta\Delta y(t) + \beta\mu y(t-1) + (1-\mu+\delta)c(t-1),$$

which is of the same form as (3.13).

Let us denote the estimate of the first coefficient in (3.13) and (3.16) by b_0 , the estimate of the second by b_1 , and so on. Then an estimate of either model may be written as

$$(3.17) \quad c(t) = b_0 + b_1\Delta y(t) + b_2y(t-1) + b_3c(t-1),$$

or, in terms of aggregate quantities,

$$(3.18) \quad C(t) = b_0P(t) + b_1Y(t) + (b_2-b_1)\frac{Y(t-1)P(t)}{P(t-1)} \\ + b_3 \frac{C(t-1)P(t)}{P(t-1)}.$$

The short-run effects on total consumption of unit changes in income and population are given by

$$SRMPC = b_1 \quad SRMPE = b_0 + (b_2-b_1)\frac{Y(y-1)}{P(t-1)} + b_3 \frac{C(t-1)}{P(t-1)}.$$

Depending on b_2 and b_1 , the short-run derivative with respect to population may be of either sign. If we set $C(t) = C(t-1)$, $P(t) = P(t-1)$, and $Y(t) = Y(t-1)$, we can examine the long-run derivatives and find

$$\text{LRMPC} = b_2/(1-b_3) \quad \text{LRMPE} = b_0/(1-b_3).$$

If an equation of the form of (3.17) has been estimated, and it is interpreted as having come from the structure summarized in (3.13), estimates of the underlying parameters can be obtained as follows, where a hat (^) denotes an estimate:

$$\begin{aligned} \hat{\alpha} &= \frac{b_0 b_1}{4(1-b_3)b_2} ; & \hat{\beta} &= \frac{b_1}{4(1-b_3)} ; \\ \hat{\gamma} &= 1-b_3; & \hat{\mu} &= \frac{b_2}{b_1} \end{aligned}$$

If, on the other hand, the Houthakker-Taylor interpretation of (3.17) is considered appropriate, the underlying parameters in (3.16) can be estimated as follows:

$$\begin{aligned} \hat{\alpha} &= \frac{b_0 b_1}{4b_2} ; & \hat{\beta} &= b_1; \\ \hat{\delta} &= b_3^{-1} + \frac{b_2}{b_1} ; & \hat{\mu} &= \frac{b_2}{b_1} \end{aligned}$$

Note that both interpretations yield the same estimate of the quarterly rate of depreciation of the stock.

We can use the lag polynomial, developed in the last chapter, to find the mean lag of equation (3.17). In terms of the lag operator, L , this equation may be written as

$$(3.17') \quad c(t) = \frac{b_0}{1-b_3L} + \frac{b_1-b_1L+b_2L}{1-b_3L} y(t).$$

If the coefficient of $y(t)$ is denoted by $G(L)$, the discussion of the last chapter should have made it clear that the mean lag is just $G'(1)/G(1)$, or $(b_2-b_1+b_1b_3)/(b_2(1-b_3))$.

In terms of the structural parameters in (3.13) the mean lag is $(1/\hat{\gamma}) - (1+\hat{\mu})$. If γ is near one, so that permanent income is always nearly equal to current income, the mean lag may well be negative. In terms of the Houthakker-Taylor parameters, the mean lag is equal to $\hat{\delta}/[\hat{\mu}(\hat{\mu}-\hat{\delta})]$, which may also be negative. Consideration of the definition of the mean lag in Chapter 2 should convince you that the mean lag can be negative only if some of the lag weights (the w_i) are themselves negative. In the case of (3.17), a negative mean lag arises if SRMPC is greater than LRMP, so that a maintained increase in y leads to a large

initial increase in c followed by a decline to the new equilibrium; during the period of the decline the lag weights are negative.

Further insight may be obtained by considering the parameter δ in the Houthakker-Taylor structure. If δ is less than zero, there is, in the Houthakker-Taylor terminology, "satiation": the more stock on hand, the less additional stock is desired at any level of income. If, on the other hand, δ is positive, we have "habit formation": the higher past consumption has been, the greater the desire for future consumption.

Houthakker and Taylor argue that satiation is likely to be observed for durables, while habit formation should be the rule for non-durables and services. If satiation prevails, we have a SRMPC above the LRMP: the mean lag is negative (and hence conveys no insight), and the mechanism at work is very much like the accelerator of investment theory. This may become a bit clearer if we write the LRMP in terms of the Houthakker-Taylor parameters as

$$\text{LRMP} = \frac{\beta\mu}{\mu-\delta}.$$

This is less than β , the SRMP, if and only if δ is negative.

It must be recognized that the equations considered here are only the most common of a host of alternative functional forms.

Both were tried for all categories of consumption, but other distributed lag mechanisms were investigated as well. We did not broaden our inquiry to consider the influence of wealth on consumption, however, in spite of the important work of Franco Modigliani and his co-authors.⁷ As mentioned in Chapter 1, we made this decision because the determinants of short-run changes in aggregate wealth are not well understood and hence it is very difficult to predict such changes in an econometric model.

3.3 THE EQUATIONS

The first equation in the consumption sector is the identity defining Y , real per capita disposable income (the $y(t)$ of the last section):

$$(3.19) \quad Y = (YDPI/LTPOP)(100./PCTOT).$$

A second identity is needed to convert estimates of per capita consumption into estimates of total consumption:

$$(3.20) \quad CTOT = (CPTOT)(LTPOP).$$

Given Y , there are two possible ways of obtaining $CPTOT$.

First, we could have one consumption function determining this quantity. In this case, the consumption sector would consist of that stochastic equation and identities (3.19) and (3.20). A second approach would involve separate equations for per capita spending in each of the main categories (CPDUR, CPND, CPHS, and CPNS), along with the following identity:

$$(3.21) \quad CPTOT = CPND + CPDUR + CPHS + CPNS.$$

Variants of this breakdown appear in many standard macro-econometric models (automobiles are sometimes explained separately from other durables, while consumer services are often not separated as we felt obliged to do), and thus our results with this approach are of interest for comparative purposes. Also, the proper level of aggregation is an unresolved issue in macro-econometric modeling, so that comparisons between the two approaches are worthwhile. Third, it will turn out that the dynamics differ enough among the major categories in (3.21) so that for some dynamic purposes only the separate equations should be used. Finally, certain fiscal policies are specific to a given production sector (e.g., durable manufacturing), and such policies can only be modeled using the disaggregated version of the consumption sector.

Both approaches were thus investigated, and we now turn to a presentation of the behavioral equations involved. We compare the estimates at the end of this section. In the next section we present yet a third variant, based on these same equations, and we discuss the choice among the three possible versions of this sector.

Aggregate Consumption Spending

The best equation for total per capita consumption spending, CPTOT, was of the form of (3.17), but without a statistically insignificant intercept:

$$(3.22) \quad CPTOT = .5997\Delta Y + .2075 Y(t-1) + .7754 CPTOT(t-1)$$

(6.72)

(2.88)

(9.85)

$R^2 = .998$
 $SE = 11.43$
 $DW = 2.37$

As discussed in Chapter 1, all equations reported in this text are ordinary least squares estimates employing data from the period 1954 I - 1969 IV. Numbers in parentheses are (absolute values of) t-statistics, SE is the standard error of the regression, and DW is the Durbin-Watson statistic.

The short-run marginal propensity to consume is .60, while

the LRMP of .924 equals the average propensity, as one would expect where the average propensity has been stable. Since the SRMP is less than the LRMP, equation (3.22) indicates net habit formation in the Houthakker-Taylor terminology. The estimated depreciation rate of the (mostly psychic) stock is 34.6% per quarter, and the mean lag is only 1.56 quarters. The LRMP is zero because there is no intercept, while the SRMP at the point of sample means is \$651.28, only 35% of the sample mean of CPTOT, \$1841.14.

Multiplying the standard error of (3.22) by the sample mean of LTPOP gives an estimated standard error in terms of CTOT of \$2.11 billion, which compares favorably with the standard errors of other aggregate consumption equations. The simulated errors are larger than this would indicate, however, as we shall see in the next section. This is inevitable in equations where lagged dependent variables appear, as they do throughout this and most other macroeconometric models.

Non-Durables

The best equation for this category was also of the form of (3.17):

$$(3.23) \quad \text{CPND} = 182.82 + .236 \Delta Y + .1534 Y(t-1) + 4266 \text{CPND}(t-1)$$

(4.95) (5.39) (4.93) (3.69)

$$\begin{aligned} R^2 &= .994 \\ SE &= 5.61 \\ DW &= 2.27 \end{aligned}$$

Multiplication of the standard error of this equation by the sample mean of LTPOP yields an estimated standard error in terms of aggregate spending of \$1.03 billion.

The Houthakker-Taylor interpretation is clearly the more appropriate here; in terms of (3.16) the structural parameter estimates from (3.23) are

$$\begin{aligned} \hat{\alpha} &= 70.5 & \hat{\beta} &= .235 \\ \hat{\mu} &= .648 & \hat{\delta} &= .075 \end{aligned}$$

The small positive estimate of δ indicates mild habit formation. Since the stock here is a purely psychic one, the estimated depreciation rate of 65% per quarter is both hard to interpret and largely irrelevant.

Using the formulae developed in the last section, we can compute the following quantities from (3.23), where all are evaluated at the point of sample means:

$$\begin{aligned} \text{SRMPC} &= .237 & \text{SRMPE} &= 381.23 \\ \text{LRMPC} &= .267 & \text{LRMPE} &= 318.84 \end{aligned}$$

Both the SRMPC and the LRMPC seem reasonable. The two population effects are both less than 45% of the mean of CPND over the sample period. The mean lag of equation (3.23) is only .20 quarters, short but not in sharp conflict with casual observation of consumer behavior.

Durables

The best equation for per capita expenditures on consumer durables was again of the same form as (3.17):

$$(3.24) \quad \text{CPDUR} = -50.18 + .2828\Delta Y + .05734 Y(t-1) + .7679 \text{CPDUR}(t-1)$$

(2.61) (4.23) (2.79) (9.38)

$$\begin{aligned} R^2 &= .988 \\ SE &= 8.56 \\ DW &= 2.14 \end{aligned}$$

In terms of aggregate spending, the estimated standard error is \$1.58 billion.

Either of the two interpretations of (3.17) presented in the last section are applicable here. In terms of equation (3.13), the structural parameter estimates from (3.24) are

$$\begin{aligned} \hat{\alpha} &= -266.24 & \hat{\beta} &= .305 \\ \hat{\mu} &= .203 & \hat{\gamma} &= .232 \end{aligned}$$

The Houthakker-Taylor structural parameter estimates are

$$\begin{array}{ll} \hat{\alpha} = -61.79 & \hat{\beta} = .283 \\ \hat{\mu} = .203 & \hat{\gamma} = -.029 \end{array}$$

Since $\hat{\delta}$ is negative, satiation is indicated, as we would expect. The estimated depreciation rate, 20.3% per quarter, seems on the high side.

As before, we can calculate the short- and long-run income and population effects at the point of means:

$$\begin{array}{ll} \text{SRMPC} = .283 & \text{SRMPE} = -286.95 \\ \text{LRMPC} = .247 & \text{LRMPE} = -216.19 \end{array}$$

The negative SRMPE is 98.6% of the sample mean of CPDUR. Since the LRMPC is below the SRMPC, the computed mean lag is negative, and there is no obvious alternative measure of the speed of adjustment.

Services

At the start of our investigations, we tried a number of equations to explain total services consumption per capita. As most other investigators had, we obtained equations in which the lagged dependent variable had a coefficient of approximately unity and was contributing most of the explanatory power, because

services consumption is highly correlated with time and hence its own lagged value. This empirical result is not consistent with the interpretation of the coefficient of the lagged dependent variable as one minus a speed of adjustment. Total per capita services were split into CPHS and CPNS to find out whether the result was due to aggregation difficulties. Housing services were, in fact, almost entirely trend, but non-housing services responded noticeably and sensibly to changes in income.⁸

The best equation for non-housing services was of the simple permanent income - partial adjustment form:

$$(3.25) \quad \text{CPNS} = \underset{(2.95)}{.02432} Y + \underset{(23.7)}{.8951} \text{CPNS} (t-1)$$

$$\begin{array}{rcl} R^2 & = & .999 \\ SE & = & 2.52 \\ DW & = & 2.10 \end{array}$$

In terms of aggregate spending, the estimated standard error is \$465 million.

The lagged dependent variable is clearly quite important here, but the mean lag of 8.53 quarters is shorter than any of our estimates for total services and thus, we believe, more realistic. The short-run and long-run derivatives evaluated at the sample means are as follows:

$$\text{SRMPC} = .024$$

$$\text{SRMPE} = 398.67$$

$$\text{LRMPC} = .232$$

$$\text{LRMPE} = 0.0$$

The SRMPE is 89% of the sample mean of CPNS, \$448.42.

As mentioned above, ~~non~~-housing services proved to be almost entirely trend. No equation with a lagged dependent variable had a significant income term or a significant intercept. A large part of this category consists of imputed rent on owner-occupied houses, but the lack of an income effect is still surprising, especially since we are considering per-capita expenditure.⁹ The following was the most sensible equation we could devise for this category:

$$(3.26) \quad [\text{CPHS}/Y] = .00426 + .9694 [\text{CPHS}(t-1)/Y(t-1)]$$

(1.30) (38.5)

$$\begin{aligned} R^2 &= .960 \\ SE &= .00109 \\ DW &= 1.48 \end{aligned}$$

The implied mean lag of this equation is an extreme 31.8 quarters. The Durbin-Watson statistic is a bit low, and the intercept is not significant at any reasonable level. Clearly this equation is far from satisfactory on both theoretical and

statistical grounds and cannot be expected to perform well beyond the period of fit, but it was the best we could find. Removing the intercept, for instance, gave the lagged dependent variable a coefficient in excess of unity. This implies that CPHS/Y will rise without limit - a nonsensical result.

Multiplying the standard error of (3.26) by the mean of Y to make it comparable with the standard errors of the other behavioral equations presented in this section yields \$2.23. Multiplying by the mean of CTPOP yields an estimated aggregate standard error of \$411 million. Both the short- and long-run population effects in (3.26) are identically zero. The short-run MPC is .130, and the LRMP is .139; both seem reasonable..

Comparison of Estimates

A comparison of the aggregate equation (3.22) with the equations for the various categories seems to favor the former. Its standard error of \$11.43 is only 60% of the total of the standard errors of equations (3.23) - (3.26).¹⁰ Perhaps the difference in performance is explained by the omission of relative prices in the equations for the individual categories; the excluded substitution effects among consumption components are automatically subsumed by the aggregate equation. We did

not experiment with relative price terms, however, for the same reason we did not investigate wealth effects: given the current state of knowledge, it is impossible to forecast relative prices with any accuracy.

In terms of income and population effects, equation (3.22) agrees fairly well with the sum of the effects in (3.23) - (3.26). The total SRMPC is .674, a bit above the estimate of .600 in (3.22), while the total LRMPC of .885 is slightly below the aggregate estimate of .924. This implies, in a rough way, that the aggregate equation embodies longer lags on average than the individual equations.¹¹

Equation (3.22) estimated a short-run population effect of \$651.28 with a LRMPE of zero. The corresponding figures obtained by adding the population effects of equations (3.23) - (3.26) were \$492.95 and \$102.05.

3.4 SIMULATION RESULTS

The first version of the consumption sector to be considered is composed of stochastic equation (3.22) and identities (3.19) and (3.20). Simulation results for this model are shown in Table 3.2. Since Y is identically determined as a function of quantities exogenous to this sector, output for this quantity

Table 3.2

Aggregated Consumption Sector
Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
1954I - 1969IV					
CTOT	347.92	3.32	.96	- .28	- .67
CPTOT	1871.41	17.88	.96	-1.67	- .74
1970I - 1970IV					
CTOT	477.08	7.00	1.46	6.37	3.82**
CPTOT	2330.33	34.10	1.46	31.09	3.84**

** Significant at 5%

Table 3.3

Dis-Aggregated Consumption Sector
Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
1954I - 1969IV					
CTOT	347.92	2.73	.83	.22	.64
CPTOT	1871.41	14.95	.83	1.17	.63
CPDUR	291.10	12.28	4.64	.19	.13
CPND	864.88	5.99	.68	- .039	- .05
CPNS	448.42	4.90	1.13	- .78	-1.28
CPHS	266.97	4.80	1.85	1.83	3.26***
1970I - 1970IV					
CTOT	477.08	6.16	1.29	5.75	4.51**
CPTOT	2330.33	30.05	1.29	28.07	4.53**
CPDUR	400.70	29.62	7.71	26.53	3.49**
CPND	1015.36	9.63	.94	-5.93	-1.35
CPNS	554.67	11.21	2.02	9.55	2.82*
CPHS	359.82	4.36	1.20	-2.31	-1.08

* Significant at 10%
 ** Significant at 5%
 *** Significant at 1%

is not shown. The RMS error of CPTOT within the sample period is 56% larger than the standard error of (3.22); a change of this magnitude is common and should not be overly alarming. In 1970, this version of the sector rather badly over-estimates consumption spending.

The second version of the consumption sector consists of stochastic equations (3.23) - (3.26) and identities (3.19) - (3.21). Results for this model are shown in Table 3.3.. Again, performance within the sample period is acceptable, though the CPHS equation shows a tendency to go off the track in this long simulation, and the sector does a bad job in 1970. It is clear that the major problem is the over-estimation of durables purchases in that year, with the largest error by far occurring in the fourth quarter. The durables equation is clearly the weakest of the four category equations.

Comparing the two sets of results, one finds that the disaggregated sector does the better job of forecasting CTOT, the most important variable. Given the standard errors of estimate of the various equations, this is somewhat surprising. What happens is that the errors from the various component equations tend systematically to cancel out. Thus while the sum of the RMS

errors for CPDUR, CPND, CPNS, and CPHS for 1970 is 54.83, the RMS error of their sum is only 30.05. This tendency was not signalled by the estimation results; simulation (or detailed examination of residuals) was necessary to uncover it.

On the basis of Tables 3.3 and 3.4, we have chosen the disaggregated version of the consumption sector for inclusion in our model. As indicated in the last section, the long-run properties of this version are not unreasonable, and it clearly has superior predictive power.

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Footnotes for Chapter 3

1. Data through 1963 were taken from U.S. Department of Commerce, Office of Business Economics [1966], while information for later years was from July issues of the Survey of Current Business. All National Income Accounts data used in this model were constructed this way.
2. Interpolation was quadratic through 1966 and linear thereafter. The extreme smoothness of the series ensured that the choice of an interpolation method had a negligible effect on the quarterly estimates.
3. The classic work of Prais and Houthakker [1955] discusses the implications of various data transformations for measurement of consumer behavior.
4. All of this is under the assumption of no secular growth in population or aggregate income. The same analysis of short- and long-run effects can be carried out under the assumption of steady growth in either or both of these quantities, but for reasonable growth rates such an analysis agrees very closely with that presented here.

5. If the rate of growth of population, r , is a constant, the coefficient of lagged consumption in (3.8) is equal to $(1-\gamma)(1+r)$, which may be solved for γ for purposes of investigating the lag distribution of that equation.
6. See Friedman [1957], Chow [1957], and Nerlove [1958] on this point.
7. See Ando and Modigliani [1963, 1969] for summary discussions. We also did not consider the role of interest rates in the consumption function; see Weber [1970] for a recent investigation.
8. The reader should examine U.S. Department of Commerce, Office of Business Economics [1958, 1966] to learn how this and other National Income Accounts series are estimated. Partly for practical reasons and partly for theoretical reasons, CPHS is almost pure trend.
9. This troublesome category was taken as exogenous to the model reported in Liebenberg et al [1966], and this course is certainly attractive. Yet, since CPHS is logically endogenous, we have felt compelled to explain it within our model.

10. The total of the standard errors is $\$18.92 = \$5.61 + 8.56 + 2.52 + 2.23$, where the last number is the sample average of Y times the standard error of (3.26). This ignores the existence of negative covariances across equations (or positive ones) that might decrease the aggregative error.
11. This correspondence is clearest in the permanent income - partial adjustment model.

CHAPTER 4

FIXED INVESTMENT SPENDING

The decision to invest in long-lived capital assets depends on expectations about the future. The formation of such expectations is exceedingly hard to model; different mechanisms may generate expectations at different times. As a consequence, neither this nor any other macroeconomic model is particularly good at characterizing fixed investment spending. Despite efforts to maintain simplicity throughout, the inherent complexity of both the concepts and dynamic properties of investment cause this chapter to be the most complicated in this volume.

Our approach to the investment sector is somewhat novel. We obtain external estimates of the distributed lags between the decision to invest and investment spending and impose these lags on our behavioral equations. Hence, statistical estimation is focused directly on the process of expectation formation. Our approach is in the spirit of a recent paper by Charles Bischoff [1970], though we have extended his formulation in some respects. Most recent work on the investment decision derives in one way or another from Jorgenson's studies [1963, 1965], and ours is no exception.

The variables employed in this sector are described below. We then explain the basic model underlying our investigation.

Chapter 2 on distributed lags should be reviewed before reading this theoretical discussion. The estimated equations are presented, and the chapter concludes with simulation experiments. An appendix to this chapter shows the derivation of the estimated order-delivery lag for producers' durable equipment as well as the construction of the user cost variables.

4.1 THE DATA

The data series used in this chapter are listed in Table 4.1. The first four investment variables were taken from the National Income Accounts as described in Section 3.1.

CDSTS is defined as the number of private non-farm housing starts, in billions of units at annual rates, times the average value per start expressed in dollars, deflated by the implicit deflator for private non-farm residential construction. A variety of apparently identical series on housing starts and cost per unit are published by government agencies. The only starts series that gave results consistent with National Accounts data was taken from Bureau of Labor Statistics Bulletin 1260 for 1953-58, from unpublished data furnished by the Bureau of the Census for 1959-62, from Table 5 of Census Bulletin C-20-67-7 for 1963-66, from Table 5 of Census Bulletin C-20-70-3 for 1967-69, and from Table 5 of Census Bulletin C-20-71-2 for 1970.¹ Quarterly averages of

Table 4.1
Variables Appearing in Chapter 4
and in the Investment Sector

(Endogenous Variables - Discussed in Chapter 3)

INFXD	Gross Private Domestic Fixed Investment (billions of constant dollars, SAAR).
INVEQ	Gross Private Domestic Investment in Producers' Durable Equipment (billions of constant dollars, SAAR).
INPL	Gross Private Domestic Investment in Non-Residential Structures (billions of constant dollars, SAAR).
INVH	Gross Private Domestic Investment in Residential Structures (billions of constant dollars, SAAR).
CDSTS	New Private Non-Farm Housing Starts (billions of constant dollars, SAAR).
STKEQ	Net Stock of Producers' Durable Equipment, End of Quarter (billions of constant dollars).
STKPL	Net Stock of Non-Residential Structures, End of Quarter (billions of constant dollars).
STKH	Net Stock of Residential Structures, End of Quarter (billions of constant dollars).
STKEQT	Transformation of STKEQ (billions of constant dollars).
STKPLT	Transformation of STKPL (billions of constant dollars).
*IN90	Smoothed Rate of Inflation (percent per year).

(Table 4.1, Continued)

*UCE	User Cost of Producers' Durable Equipment, Excluding the Impact of Inflation (percent per year).
*UCEI	User Cost of Producers' Durable Equipment, Including the Impact of Inflation (percent per year).
UCP	User Cost of Non-Residential Structures, Excluding the Impact of Inflation (percent per year).
*UCPI	User Cost of Non-Residential Structures, Including the Impact of Inflation (percent per year).
YDPID	Real Disposable Personal Income (billions of constant dollars, SAAR).

(Endogenous Variables - Determined Elsewhere in the Model)

RCB	Average Yield on Aaa-Rated Corporate Bonds (percent per year).
RCP	Average Yield on 4-6 Month Prime Commercial Paper (percent per year).
YGPP	Real Gross Private Product (billions of constant dollars, SAAR).
YDPI	Disposable Personal Income (billions of current dollars, SAAR).
PCTOT	Implicit Price Deflator for Total Consumption Expenditures (1957-59 = 100).

(Table 4.1, Continued)

*PGPP Implicit Price Deflator for Gross Private Product
(1957-59 = 100).

(Exogenous Variables)

DQ2 Seasonal Dummy Variable, Equal to 1.0 Only in Second
Quarters, Zero Otherwise.

DQ3 Seasonal Dummy Variable, Equal to 1.0 Only in Third
Quarters, Zero Otherwise.

DQ4 Seasonal Dummy Variable, Equal to 1.0 Only in Fourth
Quarters, Zero Otherwise.

*LTPOP Total Population (billions of persons).

BCPTRT Observed Corporate Profits Tax Rate (fraction).

*DITC Dummy Variable, Equal to .05 Only in Quarters When the
Investment Tax Credit Was in Force, Zero Otherwise.

RDDEP Ratio of Deductible Depreciation in deterioration

* Variable not appearing in the Investment Sector

SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual
Rates.

seasonally-adjusted monthly totals at annual rates were used.

The single usable cost series was taken from Historical Appendix A to Census Bulletin C-20-67-7 for 1953-58, from unpublished data furnished by the Bureau of the Census for 1959-62, from Appendix A of Census Bulletin C-20-67-7 for 1963-69, and from unpublished data furnished by the Census Bureau for 1970. Quarterly averages of monthly figures were used. Finally, the implicit deflator was taken from the National Income Accounts.²

Values for the three stocks at the end of 1946 were taken from Raymond W. Goldsmith [1962].³ Values for later quarters were computed according to the following identities:

$$(4.1) \quad \text{STKEQ} = (1/4) \text{INVEQ} + (1 - .148/4) \text{STKEQ}(t-1)$$

$$(4.2) \quad \text{STKPL} = (1/4) \text{INPL} + (1 - .0586/4) \text{STKPL}(t-1)$$

$$(4.3) \quad \text{STKH} = (1/4) \text{INVH} + (1 - .024/4) \text{STKH}(t-1)$$

We divide investment at annual rates by four to obtain total quarterly spending. The coefficients of the lagged stocks are equal to one minus the quarterly rate of deterioration. We assume, as the equations show, that equipment deteriorates at 14.8% per year and that plant deteriorates at 5.86% per year. For the derivation of these estimates, see the appendix to this chapter. The estimated deterioration rate of the housing stock, 2.4% per annum, was taken from the M.I.T.-Federal Reserve Board-University of Pennsylvania model.⁴ The two transformed stocks, STKEQT and

STKPLT, will be defined below; they are functions of STKEQ and STKPL.

The exponentially smoothed rate of inflation, IN90, is computed according to⁵

$$(4.4) \text{ IN90} = (4)(100)[(\text{PGPP}-\text{PGPP}(t-1))/\text{PGPP}(t-1)](.10)+(.90)\text{IN90}(t-1).$$

The factor of four converts the first term to an annual rate of inflation, the factor of 100 makes it a percentage, and the next expression is self-explanatory. The weights of .10 and .90 on current inflation and lagged IN90 indicate that IN90 changes slowly. This is in accord with the most studies of the formation of inflationary expectation in times of fairly stable prices, and IN90 was used as a measure of expected inflation in the development of this sector.⁶ The starting value for IN90 at the end of 1946 was obtained by smoothing annual changes in PGPP starting in 1929.

The user cost series listed in Table 4.1 are discussed in some detail in the appendix. They are defined by the following identities:

$$(4.5) \text{ UCE} = \left[\frac{1-\text{DITC}-(\text{RDDEP})\text{BCPTRT}}{1-\text{BCPTRT}} \right] 14.8 + \left[\frac{1-\text{DITC}-(.125)\text{BCPTRT}}{1-\text{BCPTRT}} \right] (2.55 \text{ RCB}).$$

$$(4.6) \text{ UCEI} = \text{UCE} - \left[\frac{1-\text{DITC}}{1-\text{BCPTRT}} \right] \text{IN90}.$$

$$(4.7) \text{ UCP} = \left[\frac{1-(\text{RDDEP})\text{BCPTRT}}{1-\text{BCPTRT}} \right] 5.86 + \left[\frac{1-(.125)\text{BCPTRT}}{1-\text{BCPTRT}} \right] (2.55 \text{ RCB}).$$

$$(4.8) \text{ UCPI} = \text{UCP} - \left[\frac{1}{1-\text{BCPTRT}} \right] \text{IN90}.$$

All these are explained in the second section of the appendix. The quantity (2.55 RCB) is taken as the relevant rate of discount. As the appendix discusses, the multiple of the corporate bond rate reflects both the higher cost of equity capital and prevalent debt-equity ratios. Only UCP is used in the final sector, as we shall see below.

The final endogenous variable determined in the investment sector, real disposable personal income, is defined by the following identity:

$$(4.9) \text{ YDPID} = \text{YDPI} (100/\text{PCTOT}).$$

The first two endogenous variables determined elsewhere are private interest rates obtained from quarterly averages of monthly figures taken from the Federal Reserve Bulletin. The next four variables are standard quantities taken directly from the National Income Accounts as described in Chapter 2.

LTPOP is the quarterly average of monthly figures from the Survey of Current Business. BCPTRT is simply the ratio of corporate profits tax liability to corporate profits (excluding inventory valuation adjustment), where the latter two series are taken directly from the National Income Accounts.⁷ The final two exogenous variables reflect government tax policy and are explained in detail in the appendix to this chapter.

4.2 BASIC THEORETICAL FRAMEWORK

Two basic ideas underly most discussions of investment spending.⁸ These are the accelerator - the relation between output and the capital stock - and the cost of capital. We first discuss these concepts as they relate to business investment in plant and equipment. Allowing for lags in behavior, our model of the business investment process is constructed from these key concepts. At the end of this section, we consider housing investment.

Suppose production takes place under constant returns to scale. Then for any set of prices and interest rates, the capital-output ratio that minimizes costs is independent of the level of output. Let Y be business output and let KSI be the cost-minimizing (instantaneously optimal) end-of-period capital stock. If all prices and interest rates are constant, we can write

$$(4.10) \quad KSI = v Y,$$

where v is the cost-minimizing capital-output ratio.

Net investment is defined as the change in capital stock, while gross investment is the change in the capital stock plus spending to replace worn-out units of the stock. (In the National Accounts, net investment is gross investment minus depreciation.) If all prices and interest rates are constant and if actual capital stock is always equal to desired capital stock (KSI), the level of net investment will be proportional to the change in

output. This is the simplest version of the accelerator; it serves as the basis for a number of simple analytical business cycle models.⁹

It is easy to incorporate the cost of capital into this model. The more expensive it is to have funds tied up in plant and equipment, the fewer investment projects it will be profitable to undertake. The rate of interest, a measure of the cost of funds, affects the cost of capital, which is also influenced by debt-equity mix, tax laws, expected inflation, and the durability of the capital stock. (The appendix contains a careful derivation.) Denote the cost of capital by r . Then the foregoing discussion implies that v in equation (4.10) is a declining function of r , so we can write

$$(4.10') \quad KSI = v(r) Y, \text{ with } dv/dr < 0.$$

The augmented acceleration principle still asserts that if actual capital stock is always equal to desired capital stock, the level of net investment will equal the change in KSI.

A number of modifications must be made to this basic model.¹⁰ First, it is desirable to formulate investment decisions in terms of gross investment, since reported depreciation figures may not measure actual deterioration of capital very well. Let I be gross investment spending, and assume that due to deterioration a fraction g of the capital stock is lost every period and must be

replaced. Then gross investment will be given by

$$(4.11) \quad I(t) = KSI(t) - KSI(t-1) + gK(t-1),$$

where K is the actual (end of period) capital stock. (This equation corresponds to identities (4.1)-(4.3) in the last section.)

A more serious problem with this rudimentary accelerator formulation is that it does not adequately consider the amount of capital on hand. Suppose the economy is in a deep depression, with substantial amounts of redundant labor and capital. In this situation, KSI may be well below K , so that a rise in KSI may not affect investment at all. Allowing for deterioration, the amount of capital available during the current period if no gross investment is undertaken will be $(1-g)K(t-1)$. Desired gross investment logically equals the difference between this carry-over stock and the current value of KSI . Actual gross investment will equal desired gross investment unless the latter is negative. Since this is likely to occur only in very severe depressions, we replace (4.11) by

$$(4.12) \quad I(t) = KSI(t) - (1-g)K(t-1).$$

We must now consider the incorporation of lags in the investment process into our model.¹¹ Two sorts of lags are important here. First, there is a lag between changes in the determinants

of KSI and business decisions to purchase capital goods. The second lag is the time between the decision to invest and actual investment spending; this is most sensibly thought of as the lag between orders and deliveries of investment goods. We shall analyze these two lags in turn.

KSI is that capital stock which would maximize profits given current Y and r . But capital goods will be used in the future as well as in the current period. The fact that sales are high today will lead to business investment only if sales are expected to be high in the future as well. Here, as in most econometric work, it is assumed that expectations of the future are based on past experience. Thus the stock actually preferred, which we shall call KS , will depend on current and past values of KSI. The relation between these two quantities is assumed to be given by

$$(4.13) \quad KS(t) = P(L) KSI(t),$$

where $P(L)$ is a lag polynomial reflecting expectation formation and lags in decision-making. KSI may be viewed as the true optimal stock plus independent random disturbances, so that a lag polynomial such as $P(L)$ serves to reduce the variability of the capital budgeting process by acting as a smoothing or averaging device.

Replacing KSI in (4.11) by KS , we obtain

$$(4.14) \quad I(t) = P(L) \text{ KSI}(t) - (1-g)K(t-1).$$

We shall work with the estimates of g derived in the appendix. Then, since I and K are both observable series, for estimation purposes we would probably work with (4.13) in the form

$$(4.14') \quad I(t) - (1-g)K(t-1) = P(L) \text{ v}(r) \text{ Y},$$

making use of (4.10'). The dependent variable could be calculated given an independently obtained estimate of g and alternative forms of $P(L)$ and $v(r)$ could be estimated, tested and compared.

But we are not quite ready to discuss estimation, as the order-delivery lag has yet to be reckoned with. Let $S(t)$ be investment "starts" in period t . In the case of business investment, it makes most sense to think of $S(t)$ as new orders placed in period t . As it takes some time to fill orders for capital goods, some deliveries of capital will be made in current and future periods as a result of past starts. These deliveries must be taken into account when making decisions on $S(t)$.

The discrepancy between desired and carry-over capital stock is simply $[KS(t) - (1-g)K(t-1)]$, as discussed above. Current starts are assumed to be set equal to this discrepancy minus the backlog of capital to be delivered as a result of past starts; this backlog is simply the sum of all past starts minus all past completions (expenditures). Adding current starts, the quantity

to be set equal to the discrepancy between desired and carry-over stock is

$$\begin{aligned}
 (4.15) \quad & S(t) + S(t-1) + S(t-2) + \dots - I(t-1) - I(t-2) - \dots \\
 & = (1 + L + L^2 + L^3 + \dots) [S(t) - I(t-1)] \\
 & = \frac{1}{1-L} [S(t) - I(t-1)],
 \end{aligned}$$

where the geometric series in L has been summed. Setting this quantity equal to the discrepancy between desired and carry-over capital stock and multiplying through by $(1-L)$, we obtain

$$(4.16) \quad S(t) = [KS(t) - KS(t-1)] + I(t-1) - (1-g)K(t-1) + (1-g)K(t-2).$$

This equation can be simplified by using the identity relating gross investment to changes in the capital stock,

$$(4.17) \quad K(t) = I(t) + (1-g)K(t-1),$$

to eliminate the $(1-g)K(t-2)$ term. This yields the final equation for investment starts:

$$\begin{aligned}
 (4.18) \quad & S(t) - gK(t-1) = [KS(t) - KS(t-1)] \\
 & = P(L) [KSI(t) - KSI(t-1)].
 \end{aligned}$$

One final step must be made; an observable expression for starts is required in order to obtain an equation usable for explaining gross investment. Such an expression is obtained by

writing investment as a distributed lag function of current and past starts. Thus we assume

$$(4.19) \quad I(t) = Q(L) S(t), \text{ or } S(t) = [1/Q(L)] I(t),$$

where $Q(L)$ is a known lag polynomial.¹² If all starts were finished (delivered) in one period, we would have $Q(L) = 1$. If three quarters were delivered in one period and one quarter in two periods, $Q(L)$ would equal $.75 + .25L$.

Substituting the expression for $S(t)$ from (4.19) into (4.18) and multiplying through by $Q(L)$, we obtain the desired equation involving only observables:

$$(4.20) \quad I(t) - gQ(L)K(t-1) = P(L)[Q(L)(KSI(t) - KSI(t-1))].$$

Let us now outline how equations (4.18)-(4.20) will be employed in what follows. In the case of housing, the observed series CDSTS corresponds fairly closely to the theoretical series $S(t)$. An equation like (4.19) is then used to explain housing investment, $INVH$, given CDSTS.

The instantaneously desired stock of housing should depend on per capita income, the population, and the cost of home-owning. We specify the form of the function involved and estimate its parameters along with those of $P(L)$ using (4.18).

For plant and for equipment, equation (4.20) is used. The dependent variable is computed from the $I(t)$ and $K(t)$ series, the

known constant g , and the known lag polynomial $Q(L)$. We specify the form of $v(r)$ - see (4.10') - and estimate the parameters of this function, giving predicted values of $KSI(t)$, along with those of the lag polynomial $P(L)$.

In all three cases, KSI is assumed to be a non-linear function of its determinants. Hence, simple non-linear estimation is employed, as we discuss below.

4.3 THE EQUATIONS

Let us first state the identity giving gross fixed investment as the sum of its components:

$$(4.21) \quad INF XD = INVH + INPL + INVEQ.$$

We now present the equations determining each of the components on the right-hand side of (4.21).

Housing

We must first point out that CDSTS does not measure all housing investment starts, though the correspondence is close. In the period 1960-63, in the middle of our sample, $INVH$ averaged \$23.03 billion. Of this amount, \$21.98 billion, over 95% of the total, represented new private non-farm construction, which corresponded to CDSTS.

The lag structure used by the Commerce Department to go from

housing starts to housing expenditures in their estimation of INVH is the following:¹³

$$(4.22) \quad QH(L) = .41 + .49L + .10L^2$$

The mean lag is about seven tenths of a quarter. Imposing this lag structure and using deflated disposable income and seasonal dummy variables to explain those components of INVH other than new private non-farm construction, we obtained:¹⁴

$$(4.23) \quad \begin{aligned} \text{INVH} = & 4.222 + 1.032 [.41 \text{ CDSTS} + .49 \text{ CDSTS}(t-1) + .10 \text{ CDSTS}(t-2)] \\ & (6.64) \quad (32.4) \\ & + .00415 \text{ YDPID} - .03929 \text{ DQ2} - .5105 \text{ DQ3} - .3755 \text{ DQ4} \\ & (5.37) \quad (.247) \quad (3.20) \quad (2.35) \\ & R^2 = .948 \\ & SE = .4494 \\ & DW = 1.53 \end{aligned}$$

This equation fits well, though not as well as we had anticipated. All three quarterly dummy variables were retained for completeness, though the coefficient of DQ2 is not significant, indicating little seasonal difference between the first and second quarters. No variants on this specification performed better, though several were tried.

It is now necessary to explain housing investment starts, CDSTS. We assumed that the instantaneously optimal housing stock, KSIH, when expressed in per capita terms was a function of

per capita real disposable income and the cost of capital funds according to

$$(4.24) \quad \left[\frac{\text{KSIH}}{\text{LTPOP}} \right] = c \left[\frac{\text{YDPID}}{\text{LTPOP}} \right]^a \text{RCP}^{-b}$$

The constants a and $-b$ are clearly the elasticities of KSIH with respect to real per capita disposable income and the commercial paper rate, respectively. The third constant, c , is merely a scaling factor. We follow Maisel [1965] in using the commercial paper rate rather than the more sluggish mortgage rate as a measure of the cost of funds. When mortgage money is scarce the mortgage rate often does not rise much, even though many potential borrowers are unable to find funds. The more volatile commercial paper rate captures the impact of non-price credit rationing more completely when money markets are tight.

The estimated equations were variants of (4.18). This equation has to be multiplied by four, since CDSTS is expressed at annual rates. A variety of assumptions were made about the form of $P(L)$, and a search was made to find that combination of the parameters a and b that minimized the sum of squared errors.¹⁵ We found that the value of a , so long as it was near unity, did not affect the sum of squared errors much. We, therefore, assumed a unit income elasticity by setting a equal to one and multiplying both sides of (4.24) by LTPOP. The sum of squared errors was

sensitive to the values of b , however. The best estimate was obtained with $b = .30$, although values of $.25$ and $.35$ were almost as good statistically.

Our final equation for CDSTS was

$$\begin{aligned}
 (4.25) \quad [CDSTS - .024STKH(t-1)] &= .03929 \Delta[YDPID(t-1)/RCP(t-1)^{.30}] \\
 &\quad (2.33) \\
 &+ .03792 \Delta[YDPID(t-2)/RCP(t-2)^{.30}] \\
 &\quad (2.24) \\
 &+ .9664 [CDSTS(t-1) - .024STKH(t-2)] \\
 &\quad (41.2)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .794 \\
 SE &= 1.148 \\
 DW &= 1.95
 \end{aligned}$$

The symbol Δ is the difference operator defined in Chapter 2.

The standard error of this equation is about 6.92% of the sample mean of CDSTS.

In terms of (4.18), the estimated lag polynomial here is

$$PH(L) = \left[\frac{.0336}{.07721} \right] \left[\frac{.03929L + .03792L^2}{1 - .9664L} \right]$$

The estimated mean lag is obtained as $PH'(1)$, which is equal to

$$\left[\frac{.0336}{.07721} \right] \left[\frac{(.0336)(.11513) - (.07721)(.9664)}{(.0336)^2} \right] = 30.25 \text{ quarters,}$$

or about 7.6 years. Adding the mean lag of $QH(L)$, we obtain a total mean lag in this process of 30.94 quarters or 7.7 years. Clearly this is longer than we would like, but within the framework employed in this section no remedy is obvious.

Non-Residential Structures

If production functions have constant returns to scale, doubling output will double the instantaneously optimal capital stock. Were we further to assume that the aggregate production function is Cobb-Douglas, it is easy to show that doubling the user cost would divide the optimal capital stock in half.¹⁶ Under other assumptions about the form of the production function, different elasticities of capital with respect to output and user cost can be obtained. In this study, we assume constant returns to scale, but we do not fix the elasticity of capital demand with respect to user cost. We thus write the instantaneously optimal stock of non-residential structures as either

$$(4.26) \quad \text{KSIP} = c \text{ YGPP UCP}^{-a}, \text{ or}$$

$$(4.27) \quad \text{KSIP} = c \text{ YGPP UCPI}^{-a},$$

depending upon which user cost is employed. Here c is a scaling constant, and $-a$ is the elasticity of capital demand with respect to user cost. Multiplying (4.20) by four, since INPL is expressed at annual rates, we experimented with both (4.26) and (4.27), searching for the best estimate of the user-cost elasticity $-a$ in both cases.

For the lag structure governing the time delays between starts and completions of non-residential structures, we followed Bischoff [1970] and used the lag polynomial estimated by Mayer [1958]:

$$(4.28) \quad QP(L) = .30 L + .38 L^2 + .18 L^3 + .11 L^4 + .03 L^5.$$

The mean lag is 2.2 quarters.

Various forms of $P(L)$ were examined. The user cost without an inflation factor, UCP, performed marginally better than the user cost that contained IN90, UCPI. With $a = 1$, coefficient estimates were usually absurd. Much better fits were obtained with $a = 0$, and the best estimate of a was .20. Even though this did not represent a statistically significant improvement over $a = 0$, it was decided to retain this estimate because of a strong a priori belief that investment is influenced to some extent by changes in user cost.

Our best estimate of $P(L)$ was of the simple Koyck form. We define

$$\begin{aligned} (4.29) \quad STKPLT &= .0586 QP(L) STKPL(t-1) \\ &= .0586 [.30 STKPL(t-2) + .38 STKPL(t-3) + \\ &\quad .18 STKPL(t-4) + .11 STKPL(t-5) + .03 STKPL(t-6)]. \end{aligned}$$

This quantity has no obvious interpretation; it is presented here (and in the model) merely to simplify other equations. See the derivation of equation (4.20); $STKPLT$ as defined in (4.29) corresponds to $4gQ(L)K(t-1)$ in the notation of (4.20). Making use of (4.29), we can write our best equation for investment in non-residential structures as

$$\begin{aligned}
(4.30) \quad [\text{INPL-STKPLT}] &= .1210 \left[.30 \Delta(\text{YGPP}(t-1)/\text{UCP}(t-1))^{.20} \right. \\
&\quad (3.43) \\
&\quad + .38 \Delta(\text{YGPP}(t-2)/\text{UCP}(t-2))^{.20} + .18 \Delta(\text{YGPP}(t-3)/\text{UCP}(t-3))^{.20} \\
&\quad + .11 \Delta(\text{YGPP}(t-4)/\text{UCP}(t-4))^{.20} + .03 \Delta(\text{YGPP}(t-5)/\text{UCP}(t-5))^{.20} \left. \right] \\
&\quad + .9678 [\text{INPL}(t-1) - \text{STKPLT}(t-1)] \\
&\quad (60.0)
\end{aligned}$$

$$\begin{aligned}
R^2 &= .897 \\
SE &= .5857 \\
DW &= 2.38
\end{aligned}$$

The standard error of this equation is about 3.1% of the sample mean of INPL. The mean lag of (4.30) is easily computed to be 30.06 quarters or 7.5 years. When this is added to the mean lag of QP(L), we have the result that the estimated mean lag between changes in the determinants of the optimal stock of non-residential structures and actual investment spending in this category is 32.25 quarters or 8.1 years. As before, this is a bit longer than we expected.

Producers' Durable Equipment

Investigation of this category was also based on (4.20), and we experimented with two assumptions about the optimal stock of producers' durable equipment:

$$(4.31) \quad \text{KSIE} = c \text{ YGPP UCE}^{-a}, \text{ and}$$

$$(4.32) \quad \text{KSIE} = c \text{ YGPP UCEI}^{-a}.$$

With a variety of forms of $P(L)$, the estimate of a that consistently

explained the data best was $\alpha = 0$. Again, the Cobb-Douglas assumption $\alpha = 1$ usually gave absurd results.

We estimated the lag structure between orders and spending for producers' durable equipment using monthly data for the machinery and equipment industries. The procedures and data sources are described in the appendix to this chapter. Our best estimate is the following, which has a mean lag of 2.3 quarters:

$$(4.33) \quad QE(L) = .319 + .266 L + .102 L^2 + .032 L^3 + .028 L^4 \\ + .054 L^5 + .082 L^6 + .083 L^7 + .033 L^8.$$

As before, we multiplied equation (4.20) by four since INVEQ is measured at annual rates. Again we define a variable corresponding to $4gQ(L)K(t-1)$ in the notation of (4.20) in order to simplify presentation:

$$(4.34) \quad STKEQT = .148 QE(L) STKEQ(t-1) \\ = .148 [.319 STKEQ(t-1) + .266 STKEQ(t-2) \\ + .102 STKEQ(t-3) + .032 STKEQ(t-4) \\ + .028 STKEQ(t-5) + .054 STKEQ(t-6) \\ + .082 STKEQ(t-7) + .083 STKEQ(t-8) \\ + .033 STKEQ(t-9)].$$

Our best equation for investment in producers' durable equipment has $\alpha = 0$ and the simple Koyck lag structure:

$$\begin{aligned}
 (4.35) \quad \text{INVEQ-STKEQT} &= .2369 \left[.319 \Delta \text{YGPP} + .266 \Delta \text{YGPP}(t-1) \right. \\
 &\quad (6.84) \\
 &\quad + .102 \Delta \text{YGPP}(t-2) + .032 \Delta \text{YGPP}(t-3) \\
 &\quad + .028 \Delta \text{YGPP}(t-4) + .054 \Delta \text{YGPP}(t-5) \\
 &\quad + .082 \Delta \text{YGPP}(t-6) + .083 \Delta \text{YGPP}(t-7) \\
 &\quad + .033 \Delta \text{YGPP}(t-8) \left. \right] \\
 &\quad + .9154 \left[\text{INVEQ}(t-1) - \text{STKEQT}(t-1) \right] \\
 &\quad (51.8) \\
 R^2 &= .981 \\
 SE &= .8421 \\
 DW &= 1.97
 \end{aligned}$$

The standard error of this equation is 2.32% of the mean value of INVEQ in the sample period. The mean lag of $P(L)$ as estimated by (4.35) is 10.82 quarters or 2.7 years. Adding the mean lag of $QE(L)$, we have a total mean lag in producers' durable equipment spending of 13.11 quarters or 3.3 years. This is considerably shorter than was estimated above for non-residential structures, as one would expect.

4.4 SIMULATION RESULTS

The investment sector in our model consists of twelve of the equations presented above. Identities (4.7) and (4.9) determine UCP and YDPID as functions of quantities exogenous to this sector, and identities (4.29) and (4.34) give STKPLT and STKEQT, quantities of no interest in themselves. The four behavioral equations (4.23),

(4.25), (4.30), and (4.35) determine INVH, CDSTS, INPL, and INVEQ, and identities (4.1) - (4.3) and (4.21) yield the three capital stocks and INF XD.

A dynamic simulation of this sector was run over the period 1954I - 1969IV, the period for which the behavioral equations were estimated. The results for the last eight variables mentioned just above are summarized in Table 4.2. They are not all that might be desired.

The RMS errors of the investment equations are all well above the estimated standard errors, and the RMS percentage errors are discouragingly large. Badly biased estimated of all quantities except INPL are generated.¹⁷ Even for this quantity, errors in the first half of the period are predominantly negative, resulting in a persistent downward bias in forecasts of the corresponding stock, STKPL. It would appear that, in spite of all our attention to the theory behind these equations, the dynamic structure of the investment sector has been poorly specified.

This is emphasized by the simulation experiment carried out over the four quarters of 1970 and summarized in Table 4.3. Predictions, as measured by RMS absolute and percentage errors, are much improved. Bias exists in STKH and INF XD because of the bias in INVH, but the sector does not go wildly off the track as it did in the longer sample period simulation. A comparison of Tables 4.2 and 4.3 suggests that the investment sector forecasts

Table 4.2

Investment Sector Simulation Results:1954I - 1969IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
CDSTS	16.59	2.43	14.28	-1.69	-7.70***
INVH	22.69	2.46	10.58	-1.74	-7.94***
STKH	461.41	16.86	3.39	-14.72	-14.22***
INPL	19.14	1.04	5.45	-.060	-.46
STKPL	230.10	2.66	1.19	-1.87	-7.90***
INVEQ	36.23	4.47	13.18	3.96	15.26***
STKEQ	188.90	19.03	8.99	15.76	11.72***
INFXD	78.04	3.74	4.54	2.17	5.68***

*** Significant at 1%

Table 4.3

Investment Sector Simulation Results:1970I - 1970IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
CDSTS	15.74	.87	5.01	-.47	-1.10
INVH	20.65	1.33	6.58	1.18	3.43**
STKH	552.55	.90	.16	.84	4.14**
INPL	23.08	.55	2.46	.24	.84
STKPL	284.40	.14	.049	.014	.17
INVEQ	56.15	.63	1.12	-.036	-.099
STKEQ	287.20	.12	.042	-.019	-.28
INFXD	99.82	1.47	1.48	1.44	7.80***

** Significant at 5%

*** Significant at 1%

decently in short simulations, but that it will generate systematic errors in long runs. As Thurow [1969] has emphasized, the structural representation of investment behavior turns out to be extraordinarily tough.

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Footnotes for Chapter 4

1. Non-farm figures were not available separately for 1970. We multiplied total housing starts by .9855, a constant supplied by the Bureau of the Census, to obtain estimates of non-farm starts for that year.
2. As the reader can imagine, we feel strongly that researchers in this area should not have to go through the exhausting trial and error procedure we followed in order to find the single CDSTS series that moves with INVH. The series used by the Commerce Department to construct their estimates of INVH should be published and indicated as such.
3. Goldsmith's [1962] constant dollar estimates for the three stocks at the end of 1946 were taken from pages 181 (STKEQ), 179 (STKPL), and 177 (STKH). For each of the three series, all sectors except the two government sectors were included. These estimates were converted from 1947-49 dollars to 1958 dollars by multiplying them by Goldsmith's implicit deflators for 1958, obtained by dividing his current dollar estimates of the 1958 stocks by his constant dollar estimates.
4. de Leeuw and Giamlich [1968, p. 24, equation 21].
5. This is equation (4.A.16) in the appendix to this chapter.
6. See Feldstein and Eckstein [1970] and the references there cited; a recent paper by Turnovsky [1970] takes a somewhat different view of price expectation formation.

7. This quantity is also discussed in Chapter 7.
8. For an interesting study that is, to some extent, an exception to this rule, see Thurow [1969].
9. See, for instance, Samuelson [1939] and Allen [1956, Ch. 3].
10. See Knox [1952] and Kuh [1963] for discussions.
11. The reader should be certain he has mastered Chapter 2 before attempting the remainder of this section.
12. The idea of imposing a known $Q(L)$ in this context originated with Charles Bischoff [1970].
13. See Maisel [1965].
14. The presentation of estimates in this text is explained immediately below equation (3.22).
15. Under appropriate assumptions, which do not hold here, this is a maximum-likelihood procedure. More generally, this is a straightforward extension of least squares to a non-linear model.
16. See Section 8.2, below, for a discussion of the Cobb-Douglas production function.
17. For the worst of the three investment variables, INVEQ, only one of the 64 errors is negative.

APPENDIX TO CHAPTER 4

In the first section of this appendix, we analyze the order-delivery lag for producer's durable equipment. In the second section, we derive the formulae employed to calculate the user cost of plant and of equipment, and we discuss the empirical counterparts of the theoretical constructs appearing in those relations.

The Delivery Lag in Producer's Durable Equipment

This analysis follows an article by Joel Popkin [1965]. He noted that the machinery and equipment market classification of the Bureau of the Census corresponds closely to those industries which produce durable capital equipment. The producer's durable equipment expenditures series and the machinery and equipment industries' series overlap substantially, though some differences in coverage should be noted. Producer's durable equipment includes investment in cars and trucks, which is not part of the machinery and equipment series. Unlike producer's durables, the shipment series includes exports but excludes imports of machinery and equipment. Nonetheless, the two series have moved quite similarly in the post-war period.

Seasonally-adjusted monthly data for new orders received and shipments made by the machinery and equipment industries is published by the Bureau of the Census.¹ Both series were deflated by the wholesale price index for machinery and equipment industries, furnished by the Bureau of Labor statistics.²

As in the text, we assume a distributed lag relation between orders, ϕ , and shipments, S , is as follows:

$$(4.A.1) \quad S = Q(L) \phi$$

where $Q(L)$ is a lag polynomial. Following Popkin and other writers, we assumed that $Q(L)$ is of the following form

$$(4.A.2) \quad Q(L) = w_0 + w_1 L + \dots + w_T L^T,$$

where $\sum_{i=0}^T w_i = 1$, and $w_i = b_0 + b_1 i + \dots + b_k i^k$

That is, $Q(L)$ is treated as finite with length T , and with lag weights that can be approximated by a polynomial function of the lag. This is the scheme proposed by Shirley Almon [1965] and discussed briefly in Chapter 2. It seems suitable for this problem and Almon [1967] has used it successfully on similar data. A modification of this approach has been used by Tinsley [1967], Almon [1968], and Galper and Gramlich [1968].

It became apparent that T would need to be fairly large, so estimates were fitted to the February, 1955 - December 1968 period.³ We experimented with $k = 2$ and 4 , and $T = 4, 7, 12, 15, 18, 21$, and 24 . The polynomial coefficients, the b_i in (4.A.2), were constrained so that $w_{T+1} = 0$. That is, we forced the lag weights to approach zero as i became large. Results obtained by further constraining $w_0 = 0$ were totally unsatisfactory. For both k 's, the best fitting equation assumed $T = 24$. The quadratic lag ($k=2$) exhibited a monotone decline in the w_i . In general $k=2$ is a restrictive assumption which often distorts lag estimates. With $k=4$, however, the pattern was much more interesting. The lag weights were quite large initially and declined to essentially zero by the ninth month. There was then a second, although smaller, peak in the distributed lag later on, with the weights then declining to zero again. We thus obtained a bimodal lag distribution, with the two modes corresponding, we conjecture, to standard and specially made equipment. Almost all the estimates with $k=4$ showed this pattern.

The estimate with $k=4$ represented an improvement over the less general structure implied by $k=2$; the increase in the R^2 was significant at the 5% level. The w_i corresponding to this best estimate were as follows:

$w_0 = .1863$	$w_8 = .0139$	$w_{17} = .0256$
$w_1 = .1474$	$w_9 = .0094$	$w_{18} = .0284$
$w_2 = .1143$	$w_{10} = .0073$	$w_{19} = .0302$
$w_3 = .0886$	$w_{11} = .0071$	$w_{20} = .0306$
$w_4 = .0639$	$w_{12} = .0085$	$w_{21} = .0292$
$w_5 = .0457$	$w_{13} = .0112$	$w_{22} = .0258$
$w_6 = .0316$	$w_{14} = .0145$	$w_{23} = .0200$
$w_7 = .0211$	$w_{15} = .0183$	$w_{24} = .0115$
	$w_{16} = .0221$	

Even though there are 24 lagged months represented, the mean lag is only 6.86 months, with 54.6% of shipments occurring within four months of the receipt of orders.

The sum of the weights above is 1.0107. They were normalized to sum to unity. We then obtained the quarterly weights shown in the text by assuming that the rate of orders and shipments were uniform during the quarter.

The procedure employed is best illuminated by an example. Suppose a shorter lag distribution with the following monthly weights had been estimated.

$w_0 = .4$	$w_2 = .2$
$w_1 = .3$	$w_1 = .1$

Supposing further that the rate of orders and deliveries is uniform within quarters, we reason as follows. Let S be current quarter's shipments, and let ϕ and $\phi(-1)$ be the orders placed in the current and previous quarters. The deliveries in the last month of the current quarter come from orders placed in the current quarter and the last month of the previous quarter. We can write this as

$$S/3 = .4\phi/3 + .3\phi/3 + .2\phi/3 + .1\phi(t-1)/3$$

Similarly, we have the equations for the second and first months of the quarter:

$$S/3 = .4\phi/3 + .3\phi/3 + .2\phi(t-1)/3 + .1\phi(t-1)/3$$

$$S/3 = .4\phi/3 + .3\phi(t-1)/3 + .2\phi(t-1)/3 + .1\phi(t-1)/3$$

Adding these three equations,

$$S = (2/3)\phi + (1/3)\phi(t-1),$$

so the quarterly lag weights are $(2/3)$ and $(1/3)$.

Notice that with four monthly weights, or $1 + 3N$ in general, this process comes out "even"; all orders placed in all months of the earliest quarter considered are accounted for. Our twenty-five monthly lag weights were converted into the following nine quarterly lag weights using this approach:

$w_0 = .319$	$w_3 = .032$	$w_6 = .082$
$w_1 = .266$	$w_4 = .028$	$w_7 = .083$
$w_2 = .102$	$w_5 = .054$	$w_8 = .033$

Notice that the second mode of the monthly distribution is reflected in a second mode here. The mean lag of this structure is 2.29 quarters, almost exactly the mean lag of the monthly distribution.

The User Cost of Capital

Our treatment of the user cost of capital basically follows the early work of Jorgenson [1963, 1965].⁴ There is some difference in detail, however. First, the underlying theory will be portrayed for a world without taxes. Second, this will be modified to account for corporate income taxes. Third, the operational procedures used for measuring user cost for plant and for equipment are presented.

Basic Theory Using Jorgenson's notation, we define the following quantities for a representative firm:

L = labor hired

K = effective capital stock employed

I = gross investment

Q = output = $Q(K, L)$

\bar{s} = wage rate at time zero

\bar{q} = price of capital goods at time zero

\bar{P} = price of output at time zero

r = discount rate (cost of capital funds)

μ = expected rate of increase of all prices

δ = rate of deterioration of the capital stock

Observe that we have assumed that the firm expects no change in relative prices. The following identity relates investment to changes in the effective or net capital stock:

$$(4.A.3) \quad I = \dot{K} + \delta K,$$

where $\dot{K} = dK/dt$. Thus the larger is δ , the larger gross investment must be if the effective capital stock is to be kept constant.

The firm's problem is to choose gross investment and labor inputs at all points in time so as to maximize the present discounted value of its net cash inflows. This may be expressed algebraically as

$$(4.A.4) \quad \max_{I,L} \int_0^{\infty} (\bar{P}Q - \bar{s}L - \bar{q}I) e^{-(r-\mu)t} dt = \int_0^{\infty} J(t) dt.$$

Substituting for I from (4.A.3), the Euler necessary conditions for a maximum of the integral above may be applied.⁵ These yield the following equations, which must hold at all points in time:

$$(4.A.5) \quad \partial J / \partial L = 0; \text{ or } \partial Q / \partial L = \bar{s} / \bar{P}.$$

$$(4.A.6) \quad \partial J / \partial K - d/dt (\partial J / \partial \dot{K}) = 0; \text{ or } \partial Q / \partial K = (q/P)(\delta + r - \mu).$$

Equation (4.A.5) is just the familiar condition that the marginal product of labor equal the real wage rate. Equation (4.A.6) is also a marginal productivity condition, but instead of the purchase price of capital goods, the relevant quantity is the implicit rental value or user cost. The term on the far right of (4.A.6) is the money rate of interest, (r) , minus the own-rate of interest of capital goods, $(\mu - \delta)$. This difference is the required rate of return for new gross investment.

We will not work with all terms which appear in the expression for user cost, but only with the required rate of return. There are three reasons for ignoring the relative price term. The first is that published deflators for plant and equipment are thought by many to be badly biased and crude at best. The second and related

reason is that these price indices do not properly account for technical change. Finally, the value of the model would be seriously compromised were it forced to depend on (almost inevitably) poorly specified relative price equations.

In a world without taxes, one would want required rates of return for each capital good which would differ only in the appropriate rate of deterioration employed, and to each would correspond an equation like (4.A.6).

Corporate Taxes We now modify this basic model to allow for the influence of corporate income taxes. Following Jorgenson, we define the following tax parameters:⁶

u = corporate tax rate

v = fraction of actual deterioration of capital stock that can be deducted for tax purposes

w = fraction of (rqK) that can be deducted for tax purposes

x = fraction of capital gains (on capital) that can be deducted for tax purposes

c = rate of investment credit for tax purposes

Jorgenson assumes that the firm is trying to maximize the net present value of its cash inflows. Questions of debt-equity mix and stockholder welfare are ignored. This is indeed the simplest

course to follow, and we also assume that the capital structure is given by prior considerations. By assuming that the firm is a price-taker, Jorgenson assumes no shifting of the corporate income tax, a position we adopt also.⁷

Formally, the problem facing our representative firm may be stated as follows:

$$(4.A.7) \quad \max_{I, L} \int_0^{\infty} J'(t) dt, \text{ where}$$

$$j' = J - u[\bar{P}Q - \bar{s}L - \bar{q}K(v\delta + wr - x\mu) - c\bar{q}I]e^{-(r-\mu)t}.$$

The two Euler conditions now yield the following necessary conditions for a maximum of this integral:

$$(4.A.8) \quad \partial Q / \partial L = (s/P)(1-u).$$

$$(4.A.9) \quad \partial Q / \partial K = (q/P) \frac{1-c-uv}{1-u} \delta + \frac{1-c-uw}{1-u} - \frac{1-c-ux}{1-u} \mu.$$

These are identical to the conditions derived by Jorgenson in the papers cited, except for the explicit treatment of the investment tax credit. The expression in brackets in equation (4.A.9) is the correct required rate of return. Since capital gains or losses cannot be deducted, we have $x=0$ immediately, and

we can write the required rate of return as

$$\begin{aligned}
 (4.A.10) \quad R &= \frac{1-c-uv}{1-u} \delta + \frac{1-c-uw}{1-u} r - \frac{1-c}{1-u} \mu \\
 &= WD\delta + WRr - W\mu.
 \end{aligned}$$

We now consider how best to attach values to the variables appearing in this equation. Each element in Equation (4.A.10) will be examined in turn, starting with those that would be present in a world without taxes. Then the tax parameters which cause the values of the W's to depart from unity will be evaluated.

Deterioration Rates Hall and Jorgenson [1967] estimate the rate of deterioration, δ , as 2.5 times the reciprocal of the Bulletin F lifetime.⁸ Denoting this lifetime as L, the Hall-Jorgenson procedure is equivalent to assuming that a fraction f of original effective capital remains at the end of L years, where f is given by

$$(4.A.11) \quad f = \left(1 - \frac{2.5}{L}\right)^L.$$

For the categories of capital they consider, f falls between 6.2 and 7.4 per cent. These percentages seem low, and they vary with the lifetime considered.

We calculated L's from the Hall-Jorgenson deterioration rates. We then averaged the figures for manufacturing and non-manufacturing plant and equipment, using the net stock in each sector in 1960 as weights.⁹ The resultant average lifetimes were 38.1 years for plant and 14.3 years for equipment. Assuming that 10% of original capacity remains after these periods, the rates of deterioration emerge from the following formula:

$$(4.A.12) \quad .10 = (1-\delta)^L.$$

Our calculations yielded annual rates of deterioration of 5.86% for plant and 14.8% for equipment, as shown in the text.

Cost of Capital Funds Most previous work has taken as the cost of capital, r , some measure of the rate of interest on corporate debt. Corporations raise most of their funds from internal sources or new stock issues, however, and these sources of funds must be valued at the cost of equity. In measuring this cost, we follow Gordon and Shapiro [1956]. If the dividend yield on a stock is D/P , and dividends are expected to grow at a rate g^* , the market is discounting future dividends at a rate given by

$$(4.A.13) \quad r = \frac{D}{P} + g^*.$$

A discrete analog of this formula was used to determine the cost of equity. The dividend yield used was the published quarterly series for Standard & Poor's 500 common stocks taken from various editions of the Economic Report of the President. Dividend payments in each quarter were taken from the Survey of Current Business; the series used appears as BDIV in Chapter 7. The quarter-to-quarter rate of growth of dividends was geometrically smoothed to obtain an estimate of the expected growth rate of dividends in each quarter. It was assumed that actual dividend growth from 1929 to 1930 equaled expected dividend growth, and the annual rates of growth were smoothed until 1946 I, when quarterly figures became available. From that point on, the following equation was employed, where $g(t)$ is the actual rate of growth in period t , $g^*(t)$ is the expected rate, and a is the quarterly smoothing constant.¹⁰

$$(4.A.14) \quad g^*(t) = (1-a) g(t) + a g^*(t-1)$$

We then substituted the $g^*(t)$ thus obtained and the actual dividend yield into (4.A.13) to obtain a cost of equity for each quarter. We then averaged this series over the period 1948-1966. Smoothing constants of .3, .5, .7, and .9 were employed in these computations. The computed average cost of equity was approximately

the same for all smoothing constants. Since we expect a priori that expectations about long-run rates of growth of dividends are slow to change, we used $a = .9$ in the final computations. This figure yielded an average after-tax cost of equity of 12.09% per annum from 1948 through 1966. This is 3.28 times the cost of debt, which we take as the yield on AAA corporate bonds, RCB.

It is a well-known result that if a firm raises funds in fixed proportions from two or more sources, it should use a weighted average of their costs as its discount rate, the weights being the fractions of total funds raised from each source.¹¹

From 1954 through 1966, U.S. nonform nonfinancial corporations raised 32% of funds from debt and 68% from internal sources or new stock issues.¹² The appropriate discount rate was thus calculated as the weighted average of the cost of debt and the cost of equity, which we take as 3.28 times the cost of debt, using these percentages as weights. Formally,

$$(4.A.15) \quad r = .68 (3.28 \text{ RCB}) + .32 \text{ RCB} = 2.55 \text{ RCB},$$

where RCB is the cost of debt. The cost of capital funds, r , was thus computed as 2.55 times the yield on AAA corporate bonds.

Expected Inflation To measure the rate of inflation expected by businesses, we looked at the quarter-to-quarter

changes in the implicit deflator for private product, PGPP, expressed at annual rates. We used exponential smoothing, as described above, with a quarterly smoothing constant of .9.¹³ The expected rate of inflation, μ , is thus estimated by a weighted average of current and past percentage changes in the deflator for gross private product, the weights declining as (.9). The formula for this estimate, IN90, is

$$(4.A.16) \quad IN90 = .40 [(PGPP - PGPP(t-1))/PGPP(t-1)] + .90 IN90(t-1).$$

Corporate Tax Rate The rate of corporate taxation, u , was taken as the ratio of corporate profits taxes to corporate profits, using Survey of Current Business figures for both. This variable appears explicitly in the model as BCPTRT: see Chapter 7 for more details.

Depreciation Deductions We assumed that the change in the depreciation laws in 1954 was not designed primarily to encourage investment, but rather to make the rate of depreciation allowable for tax purposes equal to the rate of deterioration. We thus take v equal to unity from the first quarter of 1954 through the second quarter of 1962.

The calculation of v for other periods, was based on the ratio of the corporate capital consumption allowance, deflated by the implicit deflator in the Survey of Current Business for fixed, non-residential investment, to the net capital stock, taken from the Survey of Current Business as described above. For the period 1948-52, this ratio was .0629, while in 1956-59 and 1963-66, it was .0755 and .0977. Hence v is .83 before 1954 and 1.29 after accelerated depreciation was enacted in 1962 II.

Interest Deductions If interest payments are taken to be $(.32) RCBqK$, from equations (4.A.7) and (4.A.15), we find that w , the ratio of interest payments (which are deductible) to rqK is .125. If changing prices were explicitly recognized, the value of w would be slightly smaller, since some borrowing in the past was undertaken to finance plant and equipment when they were cheaper, and the amount of debt outstanding will be less than $(.32)qK$. But since w is already small, this would be a very slight adjustment, so it was ignored.

Investment Tax Credit Up to this point, the discussion has applied equally to plant and equipment, with the exception of the different rates of deterioration. The investment tax credit, in effect from 1963 III - 1966 III and in 1968, applied only to

purchases of equipment. Notice that the variable c enters all the weights in (4.A.10) so we must distinguish between the weights for plant and those for equipment in the periods when the credit was in force. We follow Bischoff [1968] and let $c = .05$ for the equipment weights WDE , WRE , and WIE (see equation (4.A.20)) in the periods when the credit was in force. This variable is always zero in the computation of WDP , WRP , and WIP .

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Footnotes for Appendix to Chapter 4

1. Data for 1953-60 were taken from Table 3 of U.S. Bureau of the Census, Current Industrial Reports, Series M3-1, Manufacturer's Shipments, Inventories and Orders (Washington: Government Printing Office, 1964). Data for later years are from Table 8 of U.S. Bureau of the Census, Manufacturer's Shipments, Inventories, and Orders: 1961-1968 (Washington: Government Printing Office, 1968).
2. Results using the undeflated series were quite similar to those reported here.
3. The reason for beginning with February will become clear below.
4. In his more recent work, Jorgenson uses the concept of quasi-rents instead of user cost. This complicates the derivation somewhat, but adds generality; see Hall and Jorgenson [1967] and Bischoff [1968].
5. On the calculus of variations and the Eurler conditions, see Allen [1962, Ch. 20] and Gelfand and Fomin [1963, Ch.1].

6. This formulation does not permit us to deal with the case where investment tax credits must be deducted from the depreciation base, though a quasi-rent approach would enable us to allow for this. This situation held only from 1962 III to 1963 IV, however, and is not of great importance.
7. See Gordon [1967] for evidence on this point and Miezowski [1969] for a discussion of Gordon's and other studies.
8. This Bulletin was originally issued by the U.S. Treasury as a guideline to businesses on plant and equipment lifetimes that were acceptable for use in calculating depreciation for tax purposes.
9. See L. Grose, I. Rottenberg, and R. C. Wasson, "New Estimates of Fixed Business Capital in the United States," Survey of Current Business, 46(December, 1966): 34-40, and additional tables in Survey of Current Business, 47(December, 1967): 20-24. We employed the Constant Cost II estimates for structures and took the net stocks as computed using double declining balance depreciation.
10. The equation used for annual rates of growth before 1946 is the same as (4.A.14), except that (a) must be replaced by $(1-(1-a)^4)$.

11. See Solomon [1963].
12. See The Economic Report of the President: 1966, p.287;
1967, p.294.
13. This implies long lags in the formation of price expectations.
See Feldstein and Eckstein [1970] and the literature there
cited for evidence on this point. A recent paper by
Turnovsky [1970] points to changes in the way price
expectations are formed.

CHAPTER 5

CHANGE IN BUSINESS INVENTORIES

Our model's inventory sector is uncomplicated, since only one important endogenous variable, inventory investment, is determined here. Yet it is by no means easy to explain statistically changes in inventories.¹ The main reasons for this are that available data do not permit the kind of disaggregation that seems essential for structural modeling, and problems of valuation and price deflation are especially severe for inventories.

We first discuss briefly the data series employed in this chapter. The next section presents the basic orientation of our study, and Section 5.3 compares two alternative equations for inventory investment. As usual, the chapter concludes with some simulation results.

5.1 THE DATA

The data series used in this sector are listed in Table 5.1. YGPP and INBIN were taken directly from the National Income Accounts, as outlined in Section 3.1.

The stock series, STBIN, was constructed from INBIN; it is employed in Chapter 7. If we let K be the stock of inventories at the start of 1947I, the stock of inventories, SH , that corresponds to INBIN can be constructed as follows:

TABLE 5.1
Variables Appearing in Chapter 5
and in the Inventory Sector

(Endogenous Variables - Discussed in Chapter 5)

INBIN Change in Business Inventories (billions of
constant dollars, SAAR).

STBIN Stock of Business Inventories on Hand at the
Start of the Quarter Minus the Stock on Hand
at the Start of 1947I, Times **Four** (billions of
constant dollars).

(Endogenous Variable - Determined Elsewhere in the Model)

YGPP Real Gross Private Product (billions of constant
dollars, SAAR).

SAAR = Seasonally-Adjusted Quarterly Total Measured at
Annual Rates

$$SH(1947\ II) = K + INBIN(1947\ I)/4.$$

$$SH(1947\ III) = K + INBIN(1947\ II)/4 + SH(1947\ II)$$

$$SH(1947\ IV) = K + INBIN(1947\ III)/4 + SH(1947\ III),$$

and so on. Since we had no particularly compelling estimate of K available, we set it equal to zero. This means that the constant term in any equation involving $STBIN$ will pick up some of the effect of inventory stocks. This, of course, is no great problem. And, since the only effect would have been to multiply the coefficients of $STBIN$ by four, we did not bother to divide by four to convert $INBIN$ to a quarterly total. We thus have the identity used to calculate $STBIN$:

$$(5.1) \quad STBIN = STBIN(t-1) + INBIN(t-1),$$

with the initial condition that $STBIN = 0$ in the first quarter of 1947.

Inventories are of three kinds: finished good stocks, stocks of raw materials, and goods in process. Most formal theories of inventory holding relate only to final goods inventories, while there is reason to suspect that the other categories have basically different dynamics. Unfortunately, $INBIN$ is not broken down by type of inventory, so the three categories must be pooled together.² This is one major reason why most models of inventory behavior, ours included, have such low predictive power. Since short-run inventory fluctuations are a major source of cyclical instability, this lacunae is a source of major weakness in the formation

and execution of countercyclical government actions.³

5.2 BASIC APPROACHES

A number of inventory models use total unfilled orders as an explanatory variable, but ours does not. First, if unfilled orders appear here, they must be explained elsewhere in the model, and this is by no means an easy task. Further, if a manufacturer accumulates unfilled orders for a particular good, he is clearly not holding a finished goods inventory stock. The level of unfilled orders thus can only influence directly raw materials and goods-in-process inventories (though it may affect the finished goods stocks in other lines via expectations). Putting this variable in an equation for total inventories seems, to a certain extent, to be sweeping a real aggregation problem under the rug.⁴

Preliminary experiments with a variety of specifications attempted to use the Galper-Gramlich [1968] estimates of delivery lags to construct a series of estimated inventory changes originating in defense production. This variable never had a coefficient significant at 10%, however. Similarly, a dummy variable designed to account for the effects of steel strikes on inventory accumulation failed to perform acceptably in these experiments.

Before discussing the choice of specification further, we must define the following:

I = Inventory Investment

X = Sales (or Output)

XP = Permanent Sales (or Output)

K = Inventory Stock at End of Period

K* = Desired Inventory Stock at End of Period

Most aggregate inventory models begin with the assumption

$$(5.2) \quad K^* = \alpha XP,$$

where α is some constant and XP is some distributed lag function of current and past values of X:

$$(5.3) \quad XP = \sum_{i=0}^{\infty} w_i X(t-i); \quad w_i \geq 0 \text{ for all } i, \quad \sum_{i=0}^{\infty} w_i = 1.$$

Given (5.2) and (5.3), it remains to relate K to K*, since I is identically given by

$$(5.4) \quad I = K - K(t-1) = \Delta K,$$

where Δ is the difference operator introduced in Chapter 2.

Two extreme views of this relation are possible. The first, which seems the more common, is that $K=K^*$ at all times.⁵ On this view, production and raw materials purchases are adjusted by firms so that the desired level of inventories is maintained at all times. A diametrically opposite view holds that desired production is related to XP and that desired and actual production are equal at all times. On this view inventory holdings respond passively to changes in demand; unanticipated inventory changes occur regularly according to this interpretation and never according to the first view.

Since the distributed lag function in (5.3) has the effect of smoothing changes in X , the first view is said to propose inventory smoothing and the second to propose production smoothing as the main mechanism at work in inventory accumulation. In all probability, both mechanisms are operative.⁶

Given the data available to us, it is not possible to measure the impact of production smoothing.⁷ Thus, against our better judgement, we are forced to act as if we held the first of the two extreme views mentioned above; we consider only inventory smoothing and replace (5.2) by

$$(5.5) \quad K = \alpha X P.$$

Equations (5.3) - (5.5) can be combined to yield an estimatable inventory equation once an assumption has been made about the form of the sequence of w_i in (5.3).

If, for instance, we make the Koyck assumption

$$(5.6) \quad w_i = \gamma (1 - \gamma)^i$$

for all i , where γ is a constant between zero and one, equation (5.3) can be re-written in terms of the lag operator L discussed in Chapter 2 as

$$(5.7) \quad X P = \frac{\gamma}{1 - (1 - \gamma)L} X$$

Substituting (5.7) into (5.5), multiplying through by $[1 - (1 - \gamma)L]$, and rearranging terms, we obtain

$$(5.8) \quad I = \alpha \gamma X - \gamma K(t-1).$$

This is simply Lovell's [1961] flexible accelerator model.

An alternative derivation of (5.8) can be carried out as follows: Replace (5.5) by the assumption

$$(5.9) \quad K^* = \alpha X,$$

and assume the following dynamic stock-adjustment mechanism:

$$(5.10) \quad I = \gamma [K^* - K(t-1)].$$

According to this formulation, a fraction γ of the difference between desired inventories and those on hand at the start of the quarter is made up each quarter. Substitution of (5.9) into (5.10) yields (5.8) immediately.

Assumption (5.6) is rather restrictive, as are the equivalent assumptions (5.9) and (5.10). Another approach is to replace (5.6) by

$$(5.11) \quad w_i = 0 \quad \text{for } i > T,$$

where T is a number to be determined on the basis of sample information. Substituting (5.3) into (5.5), multiplying both sides of the resultant equation by the difference operator Δ , and employing (5.4), we obtain

$$(5.12) \quad I = \sum_{i=0}^T (\alpha w_i) \Delta X(t-i).$$

If we further assume, following Almon [1965], that

$$(5.13) \quad (\alpha w_i) = \sum_{k=0}^K b_k i^k,$$

where K is a number to be determined on the basis of sample information, we can obtain estimates of (5.12) by estimating the b_k in (5.13).⁸

In the next section we present estimates based on (5.8) and (5.12).

5.3 ALTERNATIVE ESTIMATES

The estimate which corresponds to equation (5.8) is⁹

$$(5.14) \quad \text{INBIN} = -55.50 - .1562 \text{ STBIN} + .1878 \text{ YGPP}$$

$$(8.86) \quad (7.58) \quad (8.91)$$

$$R^2 = .691$$

$$SE = 2.57$$

$$DW = 1.20$$

Note that we have used YGPP as the most sensible proxy for the variable X of the preceding section. All coefficients are highly significant; the comparatively low R^2 reflects the volatility of the dependent variable. The Durbin-Watson statistic suggests the possibility of positive serial correlation of the disturbance terms, which calls the specification into question.

Since INBIN and YGPP are quarterly flows measured at annual rates, INBIN/4 corresponds to I in the previous section, and YGPP/4 corresponds to X. Further, STBIN/4 corresponds to $K(t-1) - K_0$, where K_0 is the stock of inventories on hand at the start of 1947I. Substituting into (5.14), we obtain the following estimates of the parameters appearing in (5.8) and of K_0 :

$$\hat{\gamma} = .1562 \quad \hat{\alpha} = \frac{.1878}{.1562} = 1.202 \quad \hat{K}_0 = \frac{55.50}{2(.1562)} = 177.62$$

Recalling (5.7), the estimated mean lag is $(1 - .1562) / .1562 = 5.40$ quarters; this seems excessively long.

Our best estimate of (5.12) set $T=6$ and $K=3$; statistics were as follows:

$$\begin{aligned}
 (5.15) \quad \text{INBIN} = & .2875\Delta\text{YGPP} + .2301\Delta\text{YGPP}(t-1) + .1840\Delta\text{YGPP}(t-2) \\
 & (4.79) \qquad (7.49) \qquad (5.19) \\
 & + .1487\Delta\text{YGPP}(t-3) + .1237\Delta\text{YGPP}(t-4) \\
 & (6.83) \qquad (3.59) \\
 & + .1082\Delta\text{YGPP}(t-5) + .1019\Delta\text{YGPP}(t-g) \\
 & (3.56) \qquad (1.76)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .740 \\
 \text{SE} &= 2.379 \\
 \text{DW} &= 1.47
 \end{aligned}$$

The coefficients follow a logical, declining pattern. A straightforward regression of INBIN on the seven changes in YGPP appearing on the right-hand side of (5.15) had a comparable R^2 and standard error, but the pattern of coefficients was considerably more erratic. Smaller values of T (four or five, not smaller) also yielded similar summary statistics, but the lag weights were U-shaped (large, then small, then large again), which is somewhat implausible.

The mean lag in (5.15) is 2.27 quarters, just over half that of (5.14). About 72% of the total effect appears within the first year. Since INBIN corresponds to $4I$ in the notation of the previous section, and YGPP corresponds to $4X$, the parameter α in (5.12) is obviously estimated as the sum of the coefficients in (5.15), 1.184. This estimate agrees quite closely with that obtained from (5.14).

Equation (5.15) has a larger R^2 , smaller standard error, and better Durbin-Watson statistic than (5.14).¹⁰ The estimates of the target inventory -- YGPP ratio are roughly comparable, but the

shorter mean lag of (5.15) seems more plausible to us. For these reasons, we take (5.15) as our preferred stochastic equation for inventory investment.

This equation leaves unexplained over a quarter of the variance in inventory accumulation, and the Durbin-Watson statistic suggests some positive serial correlation of the disturbances. Few aggregate inventory equations noticeably outperform (5.15), though (see Steckler [1969]). It is difficult to imagine that much progress will be made in this important area until INBIN can be separated into its logical components.

Even then, problems will inevitably remain. INBIN is a small difference between estimates of constant dollar inventory stocks at the ends of two adjacent periods. Arbitrary accounting elements in the valuation of inventories plus errors arising from attempts to deflate current dollar stock figures place a severe limit on the attainable accuracy of constant dollar stock estimates.

5.4 SIMULATION RESULTS

The inventory sector of this model consists of identity (5.1) and stochastic equation (5.15). Since the simulated error in STBIN in any quarter is simply the sum of past errors in INBIN, little is gained by including (5.1) in simulations of this sector.

Further, since no lagged dependent variables are present in (5.15), a simulation over the sample period would merely yield the fitted values from that regression. The simulated RMS error would identically equal the standard error of the equation.

Hence, simulation results were computed only for INBIN and only for the four quarters of 1970. These results are shown in Table 5.2. The RMS error is broadly consistent with the standard error of (5.15), but there is some indication of bias present. The ratio of the RMS percentage error clearly reveals the low explanatory power of this equation (and, we hasten to add, of most of its competitors as well).

Table 5.2

Inventory Sector Simulation Results: 1970

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
INBIN	2.98	3.02	89.28	-2.50	-2.55*

*Significant at 10%

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Footnotes for Chapter 5

1. See Steckler [1969] for some evidence on this point.
2. Disaggregation by type of inventory is possible for total manufacturing and its sub-industries, but total inventory investment, which alone appears in the National Income Accounts, cannot be disaggregated.
3. For an elegant theoretical treatment of the role of inventories in cyclical fluctuations, see Metzler [1941].
4. David Belsley's [1969] excellent study of monthly two-digit manufacturing industry inventory data shows how information on orders can be employed when disaggregation is possible.
5. See the survey by Steckler [1969] and studies by Darling and Lovell [1965], Fair [1970], and Lovell [1961].
6. See Darling and Lovell [1971] for a survey of the evidence on this point.
7. Darling and Lovell [1971] also discuss this problem.
8. See Chapter 2 for a discussion of the Almon technique; it is also employed in the Appendix to Chapter 4.

9. The presentation of estimates in this text is explained immediately below equation (3.22).
10. When one or more lagged dependent variables are present in a regression equation and serial correlation of the disturbance terms occurs, the Durbin-Watson statistic is (generally) biased toward two. Since $INBIN(t-1)$ is present as part of $STBIN$, along with $INBIN(t-2)$, $INBIN(t-3)$, and so on, the real difference between the implications of the DW statistics of (5.14) and (5.15) is greater than the numbers shown indicate.

CHAPTER 6

INTERNATIONAL TRADE

The volume of international trade carried on by the United States, while large by the standards of most other nations, is small relative to U.S. Gross National Product. Thus international trade does not have much impact on the level of aggregate economic activity in this economy, and, consequently, our model has been endowed with only the most rudimentary trade sector.

6.1 THE DATA

Table 6.1 exhibits the variables employed in this sector. All quantities, except DDSTR, a dummy variable measuring the impact of dock strikes, were taken from the National Income Accounts as outlined in Section 3.1.¹

To construct DDSTR, information on the length and location of dock strikes was taken from various Bureau of Labor Statistics Bulletins beginning with No. 1184 and terminating with No. 1339, and from the U. S. Transportation Task Force, Longshore Strikes (Washington: Government Printing Office, 1970). The country was divided into four regions: New York - New Jersey, Other East Coast, Gulf Coast, and West Coast. The fraction of total trade tonnage moved through each of these was computed for the years 1957 and 1964, when there were no strikes. As the fractions from the two years were nearly identical, they were averaged for the purpose of constructing DDSTR. The weights thus obtained for the four regions

Table 6.1

Variables Appearing in Chapter 6And in the Trade Sector

(Endogenous Variables - Discussed in Chapter 6)

TIM	Imports (billions of constant dollars, SAAR).
TRBAL	Trade Balance; Equals Net Exports of Goods and Services, Exports Minus Imports (billions of constant dollars, SAAR).

(Endogenous Variables - Determined Elsewhere in the Model)

YGPP	Real Gross Private Product (billions of constant dollars, SAAR).
PGPP	Implicit Price Deflator for Gross Private Product (1957-59 = 100).

(Exogenous Variables)

TEX	Exports (billions of constant dollars, SAAR).
PTIM	Implicit Price Deflator for Imports (1957-59 = 100).
DDSTR	Dock Strike Dummy Variable

SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual Rates.

were .15, .43, .31, and .11, respectively. The value of DDSTR for each quarter was constructed by computing the fraction of the quarter that each region's ports were closed by strikes, multiplying by the weight for (importance of) that region, and adding across regions. The dummy thus represents the fraction of "normal" imports that might be held out by dock strikes. Its effect should logically be multiplicative, since multiplication of the dummy by "normal" imports gives an indication of dollar impact.

In the sample period, DDSTR is greater than .20 only in the first quarters of 1963, 1965, and 1969, though it is non-zero for fourteen quarters in the sample. It is equal to zero for all four quarters of 1970.

6.2 ESTIMATION AND SIMULATION

The first equation in the trade sector is the identity connecting exports, imports, and the trade balance:

$$(6.1) \quad \text{TRBAL} = \text{TEX} - \text{TIM}$$

The sector needs only an equation for imports, TIM, to be complete since exports are exogenous.

Clearly imports should depend on DDSTR. If the dependence is multiplicative, as argued above, the log-log form seems the most natural one. The level of economic activity should also influence imports. Both real gross private product and real disposable personal income were tried as activity variables. Finally, the ratio

of PTIM to domestic prices should affect imports. Both the implicit deflator for personal consumption and PGPP were tried as measures of the domestic price level. All equations estimated allowed for distribution lags of the Koyck form. The best equation for TIM was the following, where natural (or Naperian) logarithms have been used:²

$$\begin{aligned}
 (6.2) \quad \log(\text{TIM}) = & -1.865 + .4453 \log(\text{YGPP}) - .1421 \log(\text{PTIM/PGPP}) \\
 & (3.74) \quad (3.93) \quad (1.06) \\
 & + .7314 \log(\text{TIM}(t-1)) + .06751 \log(1.-\text{DDSTR}) \\
 & (10.3) \quad (3.16) \\
 & R^2 = .990 \\
 & SE = .0337 \\
 & DW = 1.92
 \end{aligned}$$

Even though the relative price term was not significant even at the 10% level, we retained it in the equation because it has the correct sign and seems about the right size, and because theory strongly suggests it belongs there.

The standard error given in (6.2) relates to the log of TIM, not to TIM itself. To get some idea of the implied error in TIM, we note that for X and e constants, where e is small relative to X , the following approximation holds:³

$$(6.3) \quad e \approx X [\log(X+e) - \log(X)].$$

Hence we can obtain an approximate estimate of the implied standard error of TIM by multiplying the SE given above by the sample mean of TIM, yielding \$.930 million.

The short-run income elasticity of imports is .44, while the

long-run elasticity is $.4453/(1-.7314) = 1.66$. The corresponding price elasticities are $-.14$ and $-.53$, and the mean lag of the equation is 2.72 quarters. These estimates agree fairly well with the more elaborate ones of Houthakker and Magee [1969]; our income elasticities are slightly higher and our price elasticities slightly lower (in absolute value) than theirs. As they used a different specification with different data and a different sample period, small differences in estimates are to be expected.

The term involving the dock strike dummy variable requires further explanation. Let TIM^* be the volume of imports that would take place without any dock strikes. Taking the antilogs of both sides of (6.2) then yields

$$(6.4) \quad TIM = TIM^* (1.-DDSTR)^{.06751}$$

Clearly if DDSTR were one, there would be no imports, while if DDSTR = 0, TIM would equal TIM^* . As it should, $(1.-DDSTR)$ has an exponent less than one. If only part of the country is struck, the volume of imports entering through other ports will increase to compensate. Similarly, if a dock strike ties up a given port for only part of a quarter, the port will be busier than normal at other times during the quarter. Thus if DDSTR is equal to, say, $.10$, we would expect more than 90% of TIM^* to be imported. And this is just what the fractional exponent implies. Only for large values of DDSTR will imports be noticeably disturbed.

A model consisting of equations (6.1) and (6.2) was simulated over the 1954-69 sample period and over the four quarters of 1970. The results are shown in Table 6.2.

The sector's performance in the sample period seems quite acceptable. Large relative errors are made in the trade balance, since exports and imports are nearly equal, but imports are predicted with reasonable accuracy, and there is no evidence of bias. Imports are also predicted fairly well in 1970, but the t-test shows some systematic tendency to underestimate them. Interestingly enough, though, much larger errors were made in the first two quarters of that year than in the second half.

Table 6.2

Trade Sector Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
1954 I - 1969 IV					
TIM	27.61	1.85	5.51	.079	.340
TRBAL	3.93	1.85	417.74	-.079	-.340
1970 I - 1970 IV					
TIM	49.70	1.09	2.17	-.946	-3.04*
TRBAL	2.35	1.09	51.35	.946	3.04*

* Significant at 10%

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Footnotes for Chapter 6

1. We must make a plausible but non-trivial assumption to justify treating PTIM as exogenous. Prices of imports into this country are typically set abroad, of course, but if the markets involved are not perfectly competitive, the prices charged U.S. residents may reflect economic conditions here. We assume that this is not the case, so that PTIM is determined only by conditions abroad.

Similarly, we assume that exports, TEX, are determined entirely by conditions in other countries. This is a somewhat stronger assumption than that made above, because the prices charged by exporters for their wares surely depend to some extent on prices and wages in the U.S. Still, conditions abroad are surely important and there is no obvious way to measure them. Further, to our knowledge all macroeconometric models of the U.S. treat exports as exogenous.

2. The presentation of estimates in this text is outlined below equation (3.22).
3. Expanding $\log(X+e)$ in a Taylor series, we have

$$\begin{aligned}\log(X+e) &= \log(X) + \frac{e}{X} + \frac{e^2}{2} \frac{d^2}{dx^2} \log(X+f) \\ &= \log(X) + \frac{e}{x} - \frac{1}{2} \left[\frac{e}{X+f} \right]^2 ,\end{aligned}$$

where f is some number between zero and e . If e is small relative to X , the third term is much smaller than the second, and approximation (6.3) is valid. Note that e is always underestimated by this approximation.

CHAPTER 7

INCOME DISTRIBUTION

This chapter describes the most elaborate sector of the model, consisting of 13 stochastic equations and 7 identities. The task of the income sector is simply to determine disposable personal income.

It is legitimate to ask why so many equations are needed to determine this one quantity, since many models use far fewer equations to do the job. We feel that in so doing these models neglect strategic exogenous variables that influence the difference between gross national product and disposable income and which are essential elements in a model designed to have structural and policy relevance. In particular, important policy parameters such as tax and transfer rates are often neglected to obtain a more condensed description of the determination of disposable income.

We thus felt obliged to undertake detailed modeling of the income side of the income and product accounts. This approach forces us to live with a number of less than satisfactory equations (rather than leave them implicit) but it does serve to illustrate the character of the relations which determine disposable income.

Many of the equations in this chapter have a weak theoretical basis. The reasoning behind each equation is stated, but it must be remembered that the detailed tax and accounting rules behind many of the aggregate quantities are exceedingly complex.

7.1 THE DATA

The data series appearing in this chapter are listed in Table 7.1. All of the endogenous variables determined in this sector, as well as the first three endogenous variables determined elsewhere in the model, were taken directly from the National Income Accounts as outlined in Section 3.1, except for YNIP and YPRCE. These two quantities were computed from data in the Accounts as indicated by their definitions. These definitions may be clearer if it is recalled that Gross Government Product, YGGPI, is equal to compensation of government employees.

Total unemployment, LUT, is a quarterly average of seasonally-adjusted monthly figures from Employment and Earnings. The two interest rates are quarterly averages of monthly figures taken from the Federal Reserve Bulletin. STBIN is equal to the sum of all past inventory investment, starting in 1947I. This variable is discussed at length in Chapter 5. LPEHR is an unpublished series estimated by the Bureau of Labor Statistics. The private wage rate, WRPVT, was computed according to the following identity:

$$(7.1) \quad \text{WRPVT} = \text{YPRCE}/\text{LPEHR}.$$

The first three exogenous variables are seasonally-adjusted quarterly totals expressed at annual rates taken from the National Income Accounts, and the fourth, TIME, is self-explanatory. Total population, LTPOP, was taken as the quarterly average of monthly figures from the Survey of Current Business. BCPTRT, treated as

Table 7.1

Variables Appearing in Chapter 7
and in the Income Sector

(Endogenous Variables - Discussed in Chapter 7)

YNIP	Private National Income, Equal to National Income Minus Gross Government Product (billions of current dollars, SAAR).
BCCA	Capital Consumption Allowances (billions of current dollars, SAAR).
BIBT	Indirect Business Tax and Non-Tax Liability (billions of current dollars, SAAR).
BTRF	Business Transfer Payments (billions of current dollars, SAAR).
BCPI	Corporate Profits and Inventory Valuation Adjustment (billions of current dollars, SAAR).
YPRCE	Private Compensation of Employees, Equal to Compensation of Employees Minus Gross Government Product (billions of current dollars, SAAR).
YPTOT	Proprietors' Income (billions of current dollars, SAAR).
YRENT	Rental Income of Persons (billions of current dollars, SAAR).
NETINT	Net Interest (billions of current dollars, SAAR).
BCIVA	Inventory Valuation Adjustment (billions of current dollars, SAAR).
BCP	Corporate Profits Excluding Inventory Valuation Adjustment (billions of current dollars, SAAR).
BCPT	Corporate Profits Tax Liability (billions of current dollars, SAAR).
YPERS	Personal Income (billions of current dollars, SAAR).
BDIV	Corporate Dividends (billions of current dollars, SAAR).

(Table 7.1, Continued)

GEINT	Interest Paid by Government (billions of current dollars, SAAR).
GETRFP	Government Transfer Payments to Persons (billions of current dollars, SAAR).
GRFICA	Contributions for Social Insurance (billions of current dollars, SAAR).
YCINT	Interest Paid by Consumers (billions of current dollars, SAAR).
YDPI	Disposable Personal Income (billions of current dollars, SAAR).
GRPTX	Personal Tax and Non-Tax Payments (billions of current dollars, SAAR).

(Endogenous Variables - Determined Elsewhere in the Model)

YGPPI	Gross Private Product (billions of current dollars, SAAR).
YGGPI	Gross Government Product (billions of current dollars, SAAR).
PGPP	Implicit Deflator for Gross Private Product (1957-59 = 100).
LUT	Total Unemployment (millions of persons, SA).
RCB	Average Yield on Moody's Aaa-Rated Corporate Bonds (percent per year).
RTB	Average Yield on 3 Month U.S. Treasury Bills (percent per year).
STBIN	Stock of Business Inventories on Hand at the Start of the Quarter Minus the Stock on Hand at the Start of 1974I, Times Four (billions of constant dollars).
WRPVT	Private Sector Average Wage Rate (current dollars per hour, SA).
LPEHR	Total Man-Hours Paid for in the Private Sector (billions of hours, SAAR).

(Table 7.1, Continued)

(Exogenous Variables)

GRSUB	Subsidies Less Current Surplus of Government Enterprises (billions of current dollars, SAAR).
STADIS	Statistical Discrepancy (billions of current dollars, SAAR).
WSACCR	Wage Accruals Less Disbursements (billions of current dollars, SAAR).
TIME	Time Trend, Equals 1.0 in 1954I and Rises by 1.0 Each Quarter.
LTPOP	Total Population (billions of persons).
BCPTRT	Observed Corporate Profits Tax Rate (fraction).
LRPOP	Civilian Population Aged 65 and Over (billions of persons).
GDBT	Marketable Interest-Bearing Government Debt Held by Private Investors (billions of current dollars, SA).
TVMOA	Maximum Per-Family OASDHI Benefits per Month (current dollars).
TXRSS	Employer and Employee Tax Rate for OASDHI (percent).
TMXEPE	Index of Maximum OASDHI-Taxable Earnings per Employee (1958 = 100).
TXRUB	Employer's Tax Rate for Unemployment Insurance (percent).
CGBTRT	First Bracket Federal Tax Rate on Wages and Salaries (fraction).

SA = Seasonally-Adjusted.

SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual Rates.

exogenous, is simply the ratio of BCPT to BCP.

Estimates of LRPOP, the closest we could come to the population eligible for Social Security, were furnished by the Bureau of Labor Statistics. Estimates of the next two quantities were furnished by the Brookings Institution.¹ The remaining four series were taken from Pechman [1971]. TXRSS and TMXEPE, which relate to Old-Age, Survivors, Disability, and Health Insurance (Social Security) came from Table A-6, while the last two series were taken from Tables A-7 and A-2, respectively.²

7.2 THE EQUATIONS

The equations that make up this sector will be described roughly in the order that the corresponding variables appear in the usual tracing-out of the relationship between gross national product and disposable income.

Private National Income. We begin with the identity that determines private national income, YNIP:

$$(7.2) \quad \text{YNIP} = \text{YGPPI} - \text{BCCA} - \text{BIBT} - \text{BTRF} + \text{GRSUB} - \text{STADIS}.$$

Of the quantities on the right, only BCCA, BIBT, and BTRF are determined in this sector. We now consider the corresponding stochastic equations.

Capital Consumption. Capital consumption allowances depend on the original-cost value of the capital stock, on its age composition,

and on the prevailing depreciation laws. The major change in these laws in the sample period occurred in 1962, but a variety of attempts to capture the effects of this change through dummy variables failed. Our final equation for this quantity is³

$$(7.3) \quad \begin{array}{lcl} \text{BCCA} = & .00686 \text{ YGPPI} + & .9439 \text{ BCCA}(t-1) \\ & (2.60) & (33.6) \end{array}$$

$$\begin{array}{lcl} R^2 & = & .999 \\ \text{SE} & = & .3919 \\ \text{DW} & = & 1.64 \end{array}$$

The lagged dependent variable here is designed to reflect the influence of newly-installed capital, while the income term is a proxy for the amount of newly installed capital. We should logically have replaced YGPPI by current dollar business fixed investment, but this would have required a shaky equation for the plant and equipment deflator exclusively for this limited purpose.

Indirect Business Taxes. Indirect business taxes consist of sales and property taxes. It seems sensible to relate the change in BIBT to both the change in gross private product and its level. The first term is designed to pick up the effects on sales taxes of the change in sales, while the second is related to the change in the stock of tangible assets. The remark made just above about fixed investment holds here also. The equation used is⁴

$$(7.4) \quad \begin{array}{lcl} \Delta \text{BIBT} = & .04062 \Delta \text{YGPPI} + & .00115 \text{ YGPPI}(t-1) \\ & (3.35) & (5.21) \end{array}$$

$$\begin{array}{lcl} R^2 & = & .460 \\ \text{SE} & = & .4683 \\ \text{DW} & = & 1.74 \end{array}$$

The term Δ is the difference operator introduced in Chapter 2.

The fit of equation (7.4) is quite acceptable: the R^2 is small only because the dependent variable is a first difference. In terms of BIBT, the R^2 is about .95. The standard error of \$468 million is rather small relative to the sample mean of BIBT, \$52 billion.

Business Transfer Payments. Finally, business transfer payments average only about \$2.2 billion, and they appear related to the level of economic activity. The following simple, weak, ad hoc equation is used to explain this variable:

$$(7.5) \quad \text{BTRF} = .0002445 \text{ YGPPI} + .9571 \text{ BTRF}(t-1)$$

(1.43)

(23.2)

$R^2 = .995$
 $SE = .0539$
 $DW = 2.29$

Clearly BTRF is a very smooth series, as signaled by the importance of the lagged dependent variable in this equation.

Corporate Profits and Other Income Components. Given private national income, we need to determine corporate profits and dividend payments in order to obtain personal income. Corporate profits (including inventory valuation adjustment) are calculated as a residual, by subtracting all the other components of private national income from the total. The identity used is the following:

$$(7.6) \quad \text{BCPI} = \text{YNIP} - \text{YPRCE} - \text{YPTOT} - \text{YRENT} - \text{NETINT}$$

Equations for the last four quantities on the right of this equation will now be constructed.

Private Compensation of Employees. Private compensation of employees is given by a re-arrangement of identity (7.1), the equation used to compute WRPVT:

$$(7.7) \quad YPRCE = (WRPVT)(PEHRS).$$

The private wage rate is determined in the wage-price sector (Chapter 10), and PEHRS is determined in the labor sector (Chapter 9).

Proprietors' Income. Proprietors' income is a declining share of gross private product. We explain it with an equation that involves gradual motion towards an exponentially decaying share of gross private product:

$$(7.8) \quad \log[YPTOT/YGPPI] = -.4291 - .00128 \text{ TIME} \\ (2.57) \quad (2.56) \\ +.7982 \log[YPTOT(t-1)/YGPPI(t-1)] \\ (10.0) \\ R^2 = .984 \\ SE = .0152 \\ DW = 1.66$$

The natural or Naperian logarithm is used throughout this chapter.

This equation fits well enough to permit its employment in deriving a usable corporate profit total. The trend coefficient implies that proprietors' steady-state share is declining by about 2 1/2% per year. Multiplying the standard error of (7.8) by the mean of YPTOT, we obtain an approximate standard error in terms of YPTOT of \$780 million.⁵

Rental Income. The share of gross private product accounted for by rental income is also declining. Instead of using a time

trend, as above, the equation for YRENT in the model makes this share a declining function of real per capita gross private product:

$$\begin{aligned}
 (7.9) \quad [YRENT/YGPPI] &= .8020 [YRENT(t-1)/YGPPI(t-1)] \\
 &\quad (14.9) \\
 &\quad + 17.02 [LTPOP/YGPP] \\
 &\quad (3.57)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .986 \\
 SE &= .000481 \\
 DW &= 1.07
 \end{aligned}$$

The large coefficient of the lagged dependent variable may be interpreted as reflecting long lags (slow speeds of adjustment) in the housing market, but its importance and the low Durbin-Watson signal a weak specification. The fit of this equation is more than adequate for our purposes: multiplication of the standard error of (7.9) by the sample mean of YGPPI yields an estimated standard error of YRENT of only \$254 million, less than 1.5% of the sample mean of this quantity.

It should be possible to improve on this equation by exploiting the relation between YRENT and personal consumption of housing services. But to obtain the latter variable in current dollars, we would need the corresponding implicit deflator. However, we have deliberately chosen to minimize the number of deflators appearing in the model, a matter discussed further elsewhere.

Net Interest. The equation for NETINT arose from the following model. Let D^* be the desired amount of debt by businesses. Assume that D^* is determined by

$$D^* = a + b \text{ RCB} + c \text{ YGPPI},$$

where b should be negative to reflect the impact of debt costs and c should be positive to capture the influence of the level of activity.⁷ If actual debt, D , adjusts to D^* according to

$$D - D(t-1) = \gamma[D^* - D(t-1)]$$

we can substitute and obtain

$$D = a\gamma + b\gamma \text{ RCB} + c\gamma \text{ YGPPI} + (1-\gamma)D(t-1).$$

If NETINT is assumed equal to RCB times D , this equation can be multiplied by RCB to obtain

$$\begin{aligned} \text{NETINT} &= a\gamma \text{RCB} + b\gamma (\text{RCB})^2 + c\gamma (\text{RCB YGPPI}) \\ &+ (1-\gamma) [\text{RCB NETINT}(t-1)/\text{RCB}(t-1)]. \end{aligned}$$

Unfortunately, estimates of this final equation were rather bad. Neither of the first two terms was near statistical significance. The best equation obtainable within this framework was the following:

$$\begin{aligned} (7.10) \quad \text{NETINT} &= .0001535 (\text{RCB YGPPI}) + \\ &(.902) \\ &+ .9799 (\text{RCB NETINT}(t-1)/\text{RCB}(t-1)) \\ &(32.0) \end{aligned}$$

$$\begin{aligned} R^2 &= .996 \\ SE &= .5542 \\ DW &= 1.17 \end{aligned}$$

Clearly the term involving the lagged dependent variable is doing all the work, since its t -statistic is large and so is the R^2 for the equation, while the first coefficient is not significant at the 10% level. This term is retained in the equation, however, to avoid the absurd implication that the level of net business interest

payments is unrelated to economic activity. The Durbin-Watson statistic signals mis-specification.

Personal Income, Taxes, Transfers, and Dividends. The identity relating personal income to private national income is the following:

$$(7.11) \quad \text{YPERS} = \text{YNIP} + \text{YGGPI} - \text{WSACCR} - \text{BCPI} + \text{BDIV} \\ + \text{BTRF} + \text{GEINT} + \text{GETRFP} - \text{GRFICA} + \text{YCINT}.$$

YNIP is determined by (7.2), BTRF by (7.5), BCPI by (7.6), and gross government product, YGGPI, is determined outside this sector. The small quantity WSACCR is taken as exogenous, and we now consider the remaining five terms in (7.11) in the order of their appearance.

Corporate Profits Tax (and Inventory Valuation Adjustment). Given BCPI from (7.6), the following two identities are employed to compute corporate profits tax liability, BCPT, which is employed in the dividend equation:

$$(7.12) \quad \text{BCP} = \text{PCPI} - \text{BCIVA}, \text{ and}$$

$$(7.13) \quad \text{BCPT} = (\text{BCP})(\text{BCPTRT}).$$

The corporate tax rate, BCPTRT, is treated as exogenous, but inventory valuation adjustment, BCIVA, is not. Hence we need a stochastic equation for this quantity.

BCIVA is given by the following expression, where H is the stock of business inventories, and PH is a price index constructed by the Commerce Department especially for this purpose:

$$\text{BCIVA} = - [\text{PH} - \text{PH}(t-1)]H.$$

Notice that PH is an index, not an implicit deflator. Further, we

have only STBIN as a measure of inventory stocks less an unknown constant. Using PGPP as a proxy for the true PH, we obtained the best of a rather barren series of alternative BCIVA equations:

$$(7.14) \quad BCIVA = -.3634\Delta PGPP - .00436 (STBIN\Delta PGPP)$$

(.716)
(2.61)

$$+ .3856 BCIVA(t-1)$$

(3.11)

$$R^2 = .607$$

$$SE = 1.084$$

$$DW = 1.95$$

Clearly this is somewhat flimsy. The lagged dependent variable, which has no theoretical justification, was retained because it is the main source of the equation's explanatory power. We toyed with the idea of simply treating BCIVA as exogenous, since we were unable to obtain a completely satisfactory equation for it, or even an equation that fit well. However, BCIVA is logically endogenous, and it may be important to capture the attendant decline in this quantity in simulations that involve considerable inflation, so it was retained as an endogenous variable.

Dividends. If we let P be dividend payments, P^* be equilibrium dividend payments, π be corporate pre-tax profits, and τ be the corporate profits tax rate, the model of dividend determination proposed by Lintner [1956] is

$$P^* = a(1-\tau)\pi, \text{ and}$$

$$P-P(t-1) = b[P^*-P(t-1)] + c,$$

where a and b are constants less than one, and c is greater than or

equal to zero. Darling [1957] proposed replacing after-tax profits, $(1-\tau)\pi$, by total corporate cash flow, $(1-\tau)\pi + D$, where D is corporate capital consumption allowances.⁹

Unfortunately, there is no obvious, sensible way to determine corporate capital consumption allowances as distinct from total capital consumption allowances within the national accounting framework we have adopted. These two quantities differ substantially, so the latter cannot be used as a sensible proxy for the former; in the years 1960 - 63 in the middle of our sample, corporate capital consumption allowances averaged only 59% of BCCA. Hence we are constrained to the basic Lintner specification. The intercept is not significant, and our best equation is

$$(7.15) \quad \text{BDIV} = .0242 (\text{BCP-BCPT}) + .9631 \text{BDIV}(t-1)$$

(1.74)
(31.6)

$$\begin{aligned} R^2 &= .994 \\ SE &= .3589 \\ DW &= 1.95 \end{aligned}$$

This is clearly a very weak equation; the R^2 is high because the dependent variable is largely trend. The lagged dependent variable provides virtually all the explanatory power, and the estimated mean lag of 26.1 quarters is clearly unreasonable. A one dollar increase in after-tax profits would, according to (7.15), raise current quarter dividends by only 2.4¢. The equilibrium increase in dividends is a more reasonable 65.6¢. Had we been able to use the Darling cash flow model, we might have been able to

improve this equation considerably.

Government Interest Payments. Our equation for GEINT follows the form used by Kuh [1965]. To a first approximation, the change in GEINT is given by

$$[GEINT - GEINT(t-1)] = r[GDBT - BDBT(t-1)] + GDBT[r - r(t-1)],$$

where r is the relevant interest rate. The logical proxy for r is RTB, the treasury bill rate; this yielded

$$(7.16) \quad GEINT = .1048 + .00977 \text{ RTB } \Delta GDBT + .00147 [\Delta RTB \text{ GDBT}]$$

(4.54) (2.58) (4.18)

$$\begin{aligned} R^2 &= .305 \\ SE &= .1764 \\ DW &= 1.31 \end{aligned}$$

The explanatory power of this equation is not as low as the R^2 indicates, of course, since the dependent variable is a first difference. The standard error of (7.16) is only about 2.1% of the sample mean of GEINT, and the R^2 in terms of GEINT is above .99. The large difference in R^2 's indicates that this variable is virtually pure trend.

Government Transfer Payments. Government transfer payments are determined by the following equation:

$$(7.17) \quad \begin{aligned} GETRFP &= -4.936 + 6.555 [(LRPOP)(TVMOA)] \\ &\quad (1.47) \quad (13.9) \\ &\quad + 1012. \text{ LUT} + 36.44 [\text{TIME LUT}] \\ &\quad (1.30) \quad (2.60) \end{aligned}$$

$$\begin{aligned} R^2 &= .969 \\ SE &= 2.432 \\ DW &= .360 \end{aligned}$$

The second term captures the contribution of social security payments.

In the absence of any sensible time series for unemployment compensation benefit schedules, the last two terms were inserted to pick up unemployment compensation payments, the other major component of GETRFP. It did not seem especially useful to delete the insignificant total unemployment (LUT) term and force the trend on benefits schedules through the origin or to delete the insignificant intercept term.

Because of the trend term, this equation should not be extrapolated much beyond the period of fit. Trend terms can play a valid role in econometric applications, but since they nearly always drive an equation to plus or minus infinity, their use must be restricted accordingly.

The low Durbin-Watson of (7.17) rather clearly indicates misspecification. Our (untestable) surmise is that what is causing most of the trouble is the lack of a series for unemployment benefit rates. But the equation does "explain" the sample data fairly well.

Contributions for Social Insurance. The following equation is employed for this quantity:

$$\begin{aligned}
 (7.18) \quad \text{GRFICA} = & -7.202 + .00197[(\text{TXRSS})(\text{YPRCE} + \text{YPTOT})(\text{TMXEPE})] \\
 & (7.86) \quad (12.2) \\
 & + .03027[(\text{TXRUB})(\text{YPRCE})] \\
 & (19.2)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .995 \\
 SE &= .9602 \\
 DW &= .410
 \end{aligned}$$

The major social insurance programs are social security (OASDHI) and

unemployment compensation. The second coefficient in (7.18) is a crude measure of OASDHI-taxable earnings $[(YPRCE+YPTOT)(TMXEPE)]$ times the tax rate (TXRSS); the influence of the maximum taxable earnings per individual variable, TMXEPE, would be exceedingly difficult to incorporate in a rigorous fashion. The third term in (7.18) is simply private compensation of employees times the unemployment insurance tax rate.

That this equation is mis-specified is clearly indicated by the miniscule Durbin-Watson statistic. But (7.18) explains the data well, and it is the best we could devise. If it were possible to separate OASDHI contributions from the rest of GRFICA, our predictions of total contributions could be improved, but published data do not permit it.

Interest Paid by Consumers. The model underlying the YCINT equation is essentially identical to the one that gave rise to the NETINT equation. Disposable income, YDPI, replaces YGPPI as the logical activity variable. In contrast to NETINT, the equation suggested by the theoretical development worked quite well here, with all terms significant and of the expected signs:

$$(7.19) \quad YCINT = .4924 \text{ RCB} - .2091 \text{ (RCB)}^2 + .00264 \text{ (RCB YDPI)} \\
\begin{array}{ccc}
(5.78) & (5.45) & (4.98)
\end{array} \\
+ .6796 [\text{RCB YCINT}(t-1)/\text{RCB}(t-1)] \\
(9.88)$$

$$\begin{array}{l} R^2 = .995 \\ SE = .2523 \\ DW = .840 \end{array}$$

Other than the low Durbin-Watson statistic, this equation is quite satisfactory. Relations between purchases of consumer durables, consumer debt, and YCINT have been ignored, for the usual reason that to consider them would have required estimation of an equation for the consumer durables implicit deflator.

Disposable Personal Income. The identity determining disposable personal income is

$$(7.20) \quad \text{YDPI} = \text{YPERS} - \text{GRPTX}.$$

At this point we need an equation for personal tax and non-tax payments, GRPTX.

These payments logically depend on the level of pre-tax (personal) income, its distribution across the population, and all the detailed provisions of the income tax laws. As before, we employ a simple equation to summarize these complex relationships:

$$(7.21) \quad \log(\text{GRPTX}/\text{LTPOP}) = -5.681 + .5705 \log(\text{CGBTRT}) \\ (33.2) \quad (9.88) \\ + 1.587 \log(\text{YPERS}/\text{LTPOP}) \\ (50.3)$$

$$R^2 = .990 \\ SE = .0285 \\ DW = .720$$

Even though only the first-bracket federal tax rate was employed to capture the impact of all state and federal income tax laws, the equation has considerable explanatory power; most of the total tax yield originates in this bracket. Multiplying the standard error of (7.21) by the sample mean of GRPTX, \$59.93 billion, gives an

estimated standard error in terms of this quantity of \$1.71 billion. The low Durbin-Watson clearly and not surprisingly signals misspecification, but this equation seems adequate for our purposes.

The progressive nature of the tax structure is clearly indicated by (7.21); a 1% increase in personal income per capita is estimated to yield a 1.6% increase in personal tax payments per capita. The last coefficient is significantly different from unity at more than the 1% significance level.

7.3 SIMULATION RESULTS

The income sector in this model consists of identities (7.2), (7.6), (7.7), (7.11) - (7.13), and (7.20), along with stochastic equations (7.3) - (7.5), (7.8) - (7.10), (7.14) - (7.19), and (7.21). Since (7.7) determines YPRCE identically as a function of quantities determined elsewhere in the model, this equation was not employed in computing the simulation results for this sector.

The results of a dynamic simulation over the sample period 1954I - 1969IV are summarized in Table 7.2. The mean values shown there may help give some idea of the relative importance of the unfamiliar variables discussed in this chapter.

In terms of RMS percentage errors, the outstanding failure is the weak BCIVA equation. Huge percentage errors were made in the first two quarters of 1954 and in the second quarter of 1962, when BCIVA was virtually zero. Still, the RMS error is 87% of the mean value of this variable, so our forecasts are deplorable by any

Table 7.2

Income Sector Simulation Results:

<u>Variable</u>	<u>1954 I - 1969 IV</u>				
	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
YNIP	455.47	3.02	.76	-2.82	-20.92***
BCCA	50.24	.96	2.19	.36	3.16***
BIBT	50.06	2.73	4.98	2.45	16.21***
BTRF	2.19	.14	8.32	.023	1.36
BCPI	59.91	3.47	5.58	.41	.95
YPTOT	51.33	1.21	2.40	-.050	-.33
YRENT	17.16	.48	2.86	.029	.48
NETINT	13.36	4.50	22.49	-3.20	-8.02***
YPERS	465.37	4.53	.86	-2.00	-3.90***
BCP	61.21	3.81	6.10	.42	.88
BCPT	26.99	1.66	6.09	.14	.68
BCIVA	-1.30	1.14	14000.	-.0079	-.055
BDIV	15.83	.72	4.06	-.070	-.78
GEINT	8.22	1.42	17.02	1.26	15.60***
GETRFP	32.31	2.35	6.66	#	#
GRFICA	25.75	.94	4.39	-.011	-.092
YCINT	8.83	.54	8.15	-.00049	-.073
YDPI	405.43	3.58	.85	-1.33	-3.17***
GRPTX	59.94	2.60	3.21	-.67	-2.11**

** Significant at 5%

*** Significant at 1%

Less than .0001

measure.

The performance of the NETINT and GEINT equations is also inferior; both generate large RMS errors and biased estimates. The latter presumably goes off the track because it is written in first-difference form, so the errors cumulate over time.

While YNIP is forecast with small percentage errors, it is biased downward because of the upward biases in the BCCA and BIBT equations; see equation (7.2). Estimates of BCPI are not biased, because the downward bias in YNIP is offset by the downward bias in NETINT; see equation (7.6). BCPI is forecast with sizeable RMS errors, but this is not uncommon in macro-models, since corporate profits are most often obtained as a residual. The subtraction of nearly equal numbers magnifies relative errors.

Personal Income is forecast with small errors. The downward bias in this quantity stems from the downward bias in YNIP, which is not sufficiently offset by the upward bias in GEINT; see equation (7.11). The key variable in this sector, YDPI, is also predicted accurately, though the downward bias in YPERS leads to downward bias here as well.

Two comments should be made at this point. First, note how the problems in the BCCA and BIBT equations manage to infect the entire sector. Second, one reason why YPERS and YDPI are predicted so accurately is that YPRCE, which averaged 61% of YPERS, was taken as exogenous to this sector. It remains to be seen how well we

Table 7.3

Income Sector Simulation Results:1970 I - 1970 IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
YNIP	686.78	1.61	.23	1.49	4.14**
BCCA	84.30	.43	.50	-.38	-3.28**
BIBT	92.05	1.20	1.28	-1.10	-3.99**
BTRF	3.62	.045	1.24	.018	.76
BCPI	76.48	1.07	1.38	-.84	-2.21
YPTOT	67.65	.90	1.33	.64	1.79
YRENT	22.70	.20	.87	.13	1.48
NETINT	33.45	1.73	5.20	1.58	3.97**
YPERS	801.02	12.38	1.54	-12.10	-8.06***
BCP	81.27	2.02	2.56	.073	.063
BCPT	37.45	.95	2.66	.056	.10
BCIVA	-4.80	1.69	54.88	-.92	-1.12
BDIV	25.20	.37	1.48	.33	3.23**
GEINT	14.80	.68	4.58	-.64	-4.78**
GETRFP	73.92	10.86	14.38	-10.49	-6.43***
GRFICA	57.02	2.39	4.21	2.38	15.40***
YCINT	16.98	1.57	9.03	-1.34	-2.84
YDPI	684.78	15.19	2.20	-14.37	-5.04**
GRPTX	116.25	3.75	3.26	2.26	1.30

** Significant at 5%
*** Significant at 1%

shall be able to predict YDPI in the complete model.

A dynamic simulation was also run for the four quarters of 1970, and the results are summarized in Table 7.3. In terms of relative RMS error, BCIVA still stands out. It is encouraging that the RMS percentage error of this equation is much less in 1970, with a brisk inflation, than in the earlier sample period. This is just what we had hoped would happen.

As in the 1954-69 simulation, YNIP is predicted with small relative error but significant bias, caused by the biases in the estimates of BCCA and BIBT. Here, though, both quantities are underestimated, while in the sample they were overestimated. Once again, there is an off-setting bias in NETINT, with the result that the predictions of BCPI are not significantly biased. The relative RMS error in BCPI is, in fact, considerably lower in 1970 than in the sample period.

The RMS percentage error in YPERS, on the other hand, is almost double what it was in the sample period, and this quantity is persistently underestimated. While five of the variables which enter (7.11), the identity determining YPERS, are forecast with bias and a sixth, YCINT, shows large percentage errors, a glance at the mean errors of the quantities involved signals the trouble immediately. This sector seriously and substantially underestimates GETRFP, government transfer payments to persons. Presumably a structural change occurred which was not adequately captured by equation (7.17).

The problems of trying to model complex structures with simple equations are vividly illustrated here.

The bias in YDPI and its large relative errors stem from the problems with YPERS and from clear weaknesses in the GRPTX equation. Even though YPERS was underestimated by an average of \$12.10 billion per quarter, personal tax and non-tax payments were overestimated by an average of \$2.26 billion per quarter, adding to the downward bias in prediction of disposable income. Again, some change not captured by our simple specification seems to have occurred.

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Footnotes for Chapter 7

1. GDBT is logically endogenous to the system, but available data simply did not permit us to treat it as such; see Section 11.1 for a discussion. This series is smooth enough so that this is probably not a major problem - besides, it only appears as a determinant of GEINT, a fairly unimportant quantity.
2. We employed the unweighted average of the various rates for \$0 - \$2000 of taxable income for CGBTRT.
3. The presentation of equation estimates in this text is explained immediately below equation (3.22).
4. This equation is an approximation to the first difference of an equation relating the level of BIBT to the levels of sales and of the capital stock. Since no linear trend term appears in the original specification, consistency in specification calls for suppressing the intercept here. The original intercept, of course, vanishes upon first differencing.
5. See the discussion below equation (6.2).
6. The specifications of equations (7.8) and (7.9) were strongly influenced by unpublished work of S.J. Turnovsky.
7. In reality the desired quantity of business debt depends on interest rates, costs of equity capital, the amount and composition of business assets, tax regulations, non-interest dimensions of debt instruments (maturity, repayment provisions, etc.), and expectations about the future. The strategy of

model building adopted here is typical of much of applied macroeconomics: we are obliged to compress this complicated aspect of reality in order to achieve our goals in economical fashion.

8. See Kuh [1965].
9. It may not be obvious that $(1-\tau)\pi + D$ is corporate cash flow, and this note accordingly demonstrates this. Let R be sales revenue and C be out-of-pocket costs. Then cash flow is equal to $R-C-(\text{taxes})$, while $\pi = R-C-D$. Since taxes are simply $\tau\pi$, cash flow is equal to

$$\begin{aligned}
 & R - C - (R - C - D) \\
 &= (1-\tau)(R - C) + \tau D \\
 &= (1-\tau)(R - C - D) + D \\
 &+ (1-\tau)\pi + D,
 \end{aligned}$$

as was to be shown.

CHAPTER 8

EMPLOYMENT AND UNEMPLOYMENT

Of all the policy questions that macromodels are concerned with, matters relating to employment and unemployment have the highest priority. In this chapter, we discuss the labor sector equations that generate employment, labor force, and unemployment. As matters have turned out, the first two of these can be quite sensibly explained, while the third, which is the difference between them, is subject to large percentage errors.

8.1 THE DATA

The variables appearing in this chapter are described in Table 8.1. The primary labor force and primary population are defined to include males aged 20 and over. The secondary labor force and secondary population are composed of males aged 16-19 and all females. These correspond fairly well to accepted categories except that males aged 65 and over (or perhaps 55 and over) are normally excluded from the primary labor force. It proved impossible to construct consistent quarterly labor force and population series that conformed to this definition, so older males were retained in the primary labor force.

The first six series in Table 8.1 are quarterly averages of seasonally adjusted monthly figures taken

Table 8.1
Variables Appearing in Chapter 8
and in the Labor Sector

(Endogenous Variables - Discussed in Chapter 8)

LCP	Primary Civilian Labor Force (billions of persons, SA).
LCS	Secondary Civilian Labor Force (billions of persons, SA).
LEPVT	Total Private Employment (billions of persons, SA).
LET	Total Employment (billions of persons, SA).
LUR	Unemployment Rate; Unemployment as a fraction of the Civilian Labor Force (SA).
LUT	Total Unemployment (billions of persons, SA).
LEPHR	Total Man-Hours Paid For in the Private Sector (billions of hours, SAAR).
LWKHR	Average Hours Paid for per Employee per Week in the Private Sector (hours, SA).
ER	Ratio of Civilian Employment to Civilian Population (Components, SA).

(Endogenous Variable - Determined Elsewhere in the Model)

YGPP	Real Gross Private Product (billions of constant dollars, SAAR).
------	--

Table 8.1

Variables Appearing in Chapter 8
and in the Labor Sector (Continued)

(Exogenous Variables)

LEGC	Government Civilian Employment (billions of persons).
LEGM	Government Military Employment (billions of persons).
LPPOP	Primary Population (billions of persons).
LSPOP	Secondary Population (billions of persons).
TIME	Time Trend, Equals 1.0 in 1954 I and Rises by 1.0 Each Quarter.

SA = Seasonally-Adjusted.

SAAR = Seasonally-Adjusted Quarterly Total
 Measured at Annual Rates.

from Employment and Earnings. LPEHR is an unpublished series compiled by the Bureau of Labor Statistics. The quantity LWKHR was calculated according to

$$(8.1) \quad \text{LWKHR} = \text{LPEHR}/(52 \text{ LEPVT}).$$

The variable ER, present only to facilitate writing equations, is defined below.

Gross private product in constant dollars was taken from the National Income Accounts as outlined in Section 3.1. Government civilian and military employment were taken from Employment and Earnings; quarterly averages of monthly figures were used. Data for LPPOP and LSPOP were taken from Employment and Earnings for 1964 and later years; figures for earlier periods were furnished by the Bureau of Labor Statistics. Both series refer to the total non-institutional population.

8.2 THE EQUATIONS

Behavioral equations have been estimated for LCP, LCS, LPEHR, and LWKHR. The following identities are employed to obtain the other endogenous variables determined in this sector:

$$(8.2) \quad \text{LEPVT} = \text{LPEHR}/(52 \text{ LWKHR}).$$

$$(8.3) \quad \text{LET} = \text{LEPVT} + \text{LEGC} + \text{LEGM}.$$

$$(8.4) \quad \text{LUT} = (\text{LCP} + \text{LCS}) - (\text{LEPVT} + \text{LEGC})$$

$$(8.5) \quad \text{LUR} = \text{LUT}/(\text{LCP} + \text{LCS}).$$

The first of these is simply a transformation of equation (8.1), the identity used to compute LWKHR. The second equation merely adds the various categories of employment. Total unemployment is computed in equation (8.4) by subtracting civilian employment from the civilian labor force. Finally, equation (8.5) is an obvious identity that determines the unemployment rate.

Labor Force Participation We first discuss the determination of LCP and LCS, the two components of the civilian labor force. Numerous studies have come to the conclusion that the tighter the labor market is, the more persons seek employment.¹ In particular, persons other than prime-aged males are encouraged to enter the labor force in large numbers when the market is tight. How labor market tightness should be measured is a hard empirical question. Various functions of the unemployment rate were tried, but with absolutely no success. The best measure was the ratio of civilian employment to the civilian population, which will be written as ER. This quantity is defined by the following equation:

$$(8.6) \quad ER = (LET - LEGM) / LPPOP - LSPOP - LEGM).$$

ER as defined by (8.6) was slightly superior to the same variable without the subtraction of LEGM from numerator

and denominator. Changes in ER would not be expected to affect participation immediately, so all equations included geometric distributed lags. ER and its reciprocal were treated as alternative measures of labor market tightness.

Separate equations were estimated for primary and secondary labor force participation rates. Experiments with models involving only one equation for overall participation were attempted, but the results were inferior to those presented here.

Secondary Labor Force Participation Our best equation for secondary labor force participation is the following:²

$$(8.7) \quad \text{LCS/LSPOP} = -.02275 + .0003608 \text{ TIME} + .2837 \text{ ER} \\ \quad \quad \quad (1.10) \quad \quad (5.27) \quad \quad (6.04) \\ \quad \quad \quad + .6164 [\text{LCS}(t-1)/\text{LSPOP}(t-1)] \\ \quad \quad \quad (8.92)$$

$$\begin{aligned} R^2 &= .985 \\ SE &= .00257 \\ DW &= 1.77 \end{aligned}$$

The positive time trend reflects a secular tendency toward increased participation of women in the labor force. Equation (8.7) estimates the increase in the equilibrium participation ratio to be about .38% per year. This trend obviously cannot persist indefinitely, since the participation ratio cannot exceed unity, and thus (8.7) cannot be used much beyond the period of fit.

Equation (8.7) indicates a fairly rapid response of the secondary labor force to changes in employment opportunities; the mean lag is

only 1.6 quarters. The initial response to a rise in ER of .10 is an increase in the secondary participation rate of .028, while the equilibrium increase is .074.

Primary Labor Force Participation An equation of the form of (8.7) also performed best for primary labor force participation:

$$(8.8) \quad \begin{aligned} \text{LCP/LPPOP} = & .2872 - .0003412 \text{ TIME} + .05973 \text{ ER} \\ & (3.74) \quad (3.99) \quad (2.24) \\ & + .6352 (\text{LCP}(t-1)/\text{LPPOP}(t-1)) \\ & (6.90) \end{aligned}$$

$$\begin{aligned} R^2 & .989 \\ SE & .00185 \\ DW & = 1.72 \end{aligned}$$

The negative trend term presumably shows the tendency toward earlier retirement; the estimated decline in the steady-state primary labor force participation rate is about .37%. Again, this trend cannot be expected to hold much beyond the period of fit. This trend has the same magnitude as and opposite sign to the estimate from the secondary participation equation, so the overall participation rate shows a much smaller secular rate of change than either of its components.

Equation (8.8) indicates a slightly slower response of the primary labor force than the secondary labor force to changes in employment conditions; the mean lag is 1.7 quarters. We would expect the primary labor force to respond more slowly, since its

members are more permanently attached to the labor force .

The initial response to a rise in ER of .10 is an increase in the primary participation rate of .006, while the equilibrium rise is .016. Both these are considerably smaller than the corresponding estimates from (8.7). This indicates, as one would expect, that the primary participation rate is more stable than the secondary rate and responds more weakly to changes in employment opportunities.

Demand for Labor

Having considered the supply of labor just above, we now turn to the determination of the demand for labor which is measured by total employment. We first determine the demand for manhours in the private sector, LPEHR, in a fairly straightforward fashion. Employing this estimate, we present an equation for average weekly hours paid for per worker, LWKHR. Identity (8.2) is then employed to obtain total private employment, LEPVT.

Manhours Demand The fundamental labor input into the production process is manhours, measured by LPEHR. Our equation for this quantity follows closely the approach established by Nield [1963], Wilson and Eckstein [1964], Brechling [1965], Kuh [1965a, 1965b, 1966], Black and Russell [1969], and Black and Kelejian [1970], though the last-named article is to some extent a departure from tradition.

It is reasonably to assume that changes in output will not be

reflected immediately in changes in manhours paid for, since non-production workers, who are assumed to work a fixed number of hours per week, will not be hired or fired in response to changes in demand that their employers believe might be transitory. Further, even for production workers, hours worked generally fall short of hours employed; some increase in the former can take place without having to affect the latter.³

As the relation between aggregate output and factor inputs, we assume an aggregate production function of the popular Cobb-Douglas form.⁴

$$(8.9) \quad Y = a (MHW)^b K^c e^{dt}, \quad \text{with } a, b, c, d > 0,$$

where Y is gross private output, MHW is manhours worked, K is capital services utilized by the private sector, and the last term captures increases in factor productivity.⁵ We then add a (logarithmic) first-order adjustment mechanism relating hours worked, MHW , to hours paid for, MH :

$$(8.10) \quad MH = [f MHW]^g [MH(t-1)]^{1-g}, \quad \text{with } f, g > 0 \text{ and } g > 1.$$

Substituting from (8.10) for MHW in (8.9) and solving for MH , we obtain the basic specification explored:

$$(8.11) \quad MH = A Y^{\alpha\gamma} K^{-\eta\gamma} e^{-\beta\gamma t} [MH(t-1)]^{1-\gamma}, \quad \text{where}$$

$$A = f^g a^{-g/b}, \quad \alpha = 1/b, \quad \gamma = g, \quad \eta = c/b, \quad \text{and } \beta = d/b.$$

Taking (natural) logarithms of both sides of (8.11) yields an equation linear in the unknown parameters.

Various functions of this sort were tried, some with more complex lag structures. In none were capital stock variables significant.⁶ There are at least three possible explanations for this. First, our capital stock estimates may be excessively crude, though they are not noticeably weaker than most others. Second, the quantity which should appear in our equations is capital services utilized, not capital stock on hand. To the extent that the rate of utilization of the stock varies cyclically, the latter is a poor proxy for the former. Finally, the Cobb-Douglas function may exaggerate the short-run substitution possibilities between capital and labor.

Our demand function for manhours finally turned out to be

$$\begin{aligned}
 (8.12) \quad \log(\text{LPEHR}) = & \underset{(9.00)}{-.00237} \text{ TIME} + \underset{(9.40)}{.3297} \log(\text{YGPP}) \\
 & + \underset{(13.9)}{.5973} \log[\text{LPEHR}(t-1)]
 \end{aligned}$$

$$\begin{aligned}
 R^2 & .986 \\
 SE & .00581 \\
 DW & = 1.27
 \end{aligned}$$

The negative time trend reflects both technical progress and the secular increase in capital per worker. The mean lag in (8.12) is only 1.5 quarters, implying rather rapid adjustment of manhours paid for to changes in output. The equation fits well, though the Durbin-Watson is low. Multiplication of the standard error of

(8.12) by the sample mean of LPEHR yields an estimated standard error of that quantity of about 759 million manhours.⁷

The short-run elasticity of manhour demand with respect to output is .33, while the long-run elasticity is .82. In terms of our basic equation (8.9), this implies that b , the elasticity of output with respect to labor input, is greater than one. This is rather surprising. An alternative interpretation is, of course, that we have failed to include a good measure of capital services and have, therefore, missed much of the impact of capital deepening. In any case, equation (8.12) is the best we have been able to devise, resembles other such aggregate relations, explains the data quite well, and possesses reasonable coefficients for the most part.

Weekly Hours Some of the information in equation (8.12) will be utilized in the estimated equation for LWKHR, average weekly hours.⁸ Let M be employment, H be average hours per employee, Y be gross private product and set t equal to TIME, as before. The model underlying (8.12) may be written compactly as

$$(8.13) \quad [MH] = AY^{\alpha\gamma} e^{-\beta\gamma t} [M(t-1) H(t-1)]^{1-\gamma}$$

Suppose that the demand for employees is given by the following quite similar model:

$$(8.14) \quad M = BY^{a\delta} e^{-b\delta t} M(t-1)^{1-\delta}.$$

Since one expects employment to adjust more slowly than man-hours (people work overtime when demand rises suddenly), γ should

be larger than δ . (Notice that both γ and δ must fall between zero and one.) Similarly, since the average work week has been secularly falling, the demand for employees must be dropping off more rapidly than the demand for manhours; thus β should be larger than b . Finally, it does not seem unreasonable for α to be equal to a , though a priori this seems the weakest of the three assertions.

Dividing the second equation above by the first and taking logarithms, we obtain

$$(8.15) \quad \log[H/H(t-1)] = \log(A/B) + (\alpha\gamma - a\delta) \log(Y) + (b\delta - \beta\gamma)t \\ + (-\gamma) \log[H(t-1)] + (\delta - \gamma) \log[M(t-1)].$$

The last three terms should have negative coefficients, according to our a priori beliefs, while if a is not much larger than α , the coefficient of $\log(Y)$ should be positive.

When this model was estimated, we obtained

$$(8.16) \quad \log[LWKHR/LWKHR(t-1)] = 1.110 + .1634 \log(YGPP) - .00197 \text{ TIME} \\ (2.27) \quad (6.47) \quad (6.30) \\ - .5880 \log[LWKHR(t-1)] - .04371 \log[LEPVT(t-1)] \\ (5.47) \quad (.864) \\ R^2 = .484 \\ SE = .00397 \\ DW = 1.89 \\ SSR = .0009296$$

The quantity SSR is the sum of squared residuals, which we employ below. All coefficients in (8.16) have the expected signs, and all except the last are significant. The coefficient of $\log[LWKHR(t-1)]$ gives an estimate of γ of .5880, as against $\gamma = .4027$ estimated by (8.12).

Consider (8.15) again, and suppose that we have some estimate of γ , call it $\hat{\gamma}$, from elsewhere. If $\hat{\gamma} = \gamma$, equation (8.15) can be written as

$$(8.17) \quad \log(H) - (1 - \hat{\gamma}) \log[H(t-1)] = \log(A/B) + (\alpha\gamma - a\delta) \log(Y) \\ + (b\delta - \beta\gamma)t \\ + (\delta - \gamma) \log[M(t-1)].$$

By imposing the constraint $\gamma = \gamma$, we have reduced by one the number of coefficients to be estimated. We imposed the constraint $\gamma = .4027$ from (8.12) on (8.16) and obtained

$$\begin{aligned}
 (8.18) \quad \log(\text{LWKHR}) - .5973 \log[\text{LWKHR}(t-1)] &= .3780 + .1464 \log(\text{YGPP}) \\
 &\quad (1.55) \quad (6.20) \\
 &\quad - .00151 \text{ TIME} - .09232 \log[\text{LEPVT}(t-1)] \\
 &\quad (8.95) \quad (2.16) \\
 R^2 &= .755 \\
 \text{SE} &= .00403 \\
 \text{DW} &= 2.24 \\
 \text{SSR} &= .0009762
 \end{aligned}$$

The striking change in the R^2 is almost entirely due to the change in dependent variable; compare the standard errors.

Using the F-test based on the SSR's of (8.16) and (8.18), we can test the validity of the restriction $\gamma = .4027$ imposed in the latter equation.⁹ The F-statistic with one and 59 degrees of freedom (there are 65 observations and four parameters fitted) is

$$\frac{(9762 - 9269)/1}{9269/59} = 3.14.$$

This does not signal rejection of the hypothesis $\gamma = .4027$ at the 5% level, so it is retained.

We now consider the third of our assertions below (8.14), that a should equal α . Given an estimate of α , call it $\hat{\alpha}$, if $a = \alpha = \hat{\alpha}$, (8.17) may be re-written as

$$\begin{aligned}
 (8.19) \quad \log(\text{H}) - (1-\gamma)\log[\text{H}(t-1)] &= \log(\text{A/B}) + (b\delta - \beta\gamma)t \\
 &\quad + (\gamma - \delta)[\hat{\alpha}\log(\text{Y}) - \log[\text{M}(t-1)]].
 \end{aligned}$$

Table 8.2

Labor Sector Simulation Results:

<u>Variable</u>	<u>1954 I - 1969 IV</u>				
	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
LCP	.044	.00012	.29	#	.15
LCS	.027	.00031	1.14	#	.80
LPEHR	130.65	1.13	.87	.073	.51
LWKHR	40.73	.16	.40	.0017	.08
LEPVT	.062	.00043	.68	#	.63
LET	.071	.00043	.59	#	.63
LUT	.0035	.00031	9.13	#	.06
LUR	.049	.0041	8.83	#	.02

#Less than .00005

As before, we have imposed one constraint on the coefficients and thus reduced their number by one. Imposing the restriction

$a = \alpha = .8184$ from (8.12) on (8.18), we estimated

$$\begin{aligned}
 (8.20) \quad \log(\text{LWKHR}) - .5973 \log[\text{LWKHR}(t-1)] &= .06220 - .00131 \text{ TIME} \\
 &\quad (.265) \quad (7.94) \\
 &+ .1875 [.8184 \log(\text{YGPP}) - \log[\text{LEPVT}(t-1)]] \\
 &\quad (6.13)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .718 \\
 SE &= .00429 \\
 DW &= 2.04 \\
 SSR &= .001125
 \end{aligned}$$

The F-statistic with one and 60 degrees of freedom corresponding to the restriction imposed in (8.20) is

$$\frac{(11250 - 9762)/1}{9762/60} = 9.15.$$

This calls for rejection of the restriction at better than the 1% level.

Thus (8.18) is our preferred LWKHR equation. The negative coefficient of $\log(\text{LEPVT}[t-1])$ confirms the conjecture that γ is greater than δ . Multiplying the standard error of (8.18) by the mean of LWKHR, we obtain an approximate standard error of .164 hours per week.

8.3 SIMULATION RESULTS

The labor sector consists of identities (8.2) - (8.5) and stochastic equations (8.7), (8.8), (8.12), and (8.18). Identity (8.6) for ER merely simplifies presentation, so no simulation results for this quantity are presented.

A dynamic simulation of this sector was performed over the period of fit, 1954 I - 1969 IV, and a summary of the results is shown in Table 7.2. None of the variables go off the track, as the uniformly tiny t-statistics and small mean errors indicate. All the RMS percentage errors are miniscule, except for those in LUT and LUR.

It is clear why these variables are not forecast especially well. Both the labor force and civilian employment are forecast with modest errors. Since these two quantities are nearly equal, when we subtract to obtain unemployment, the small relative errors in employment and labor force induce errors in LUT which are large relative to that quantity. These then produce large relative errors in LUR. Thus, either far superior estimates of LCP, LCP, and LEPVT or an entirely different approach would be required to noticeably improve our ability to track LUT and LUR.

A dynamic simulation of this sector over the four quarters of 1970 was also performed, and the results are shown in Table 8.3. In terms of RMS percentage errors, the labor force equations performed just slightly worse than in the sample period. Both equations show significant bias, however, and the total labor force is consistently underestimated.

The LPEHR AND LWKHR equations perform acceptably, but much worse than they had in the sample period. There is no evidence of bias in either variable, but the percentage over-statement of LWKHR is consistently larger than that of LPEHR, leading to a significant downward bias in private (and thus in total) employment.

To some extent the downward biases in the labor force and in employment offset one another, so that LUT and LUR forecast without noticeable bias.¹⁰ In fact, the RMS percentage errors in these quantities are lower than they were in the sample period.

Table 8.3

Labor Sector Simulation Results:

<u>Variable</u>	<u>1970 I - 1970 IV</u>				
	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
LCP	.047	.00020	.43	-.00020	-9.52***
LCS	.036	.00065	1.81	-.00060	-4.16**
LPEHR	136.40	5.16	3.95	3.55	1.64
LWKHR	37.89	1.73	4.73	1.35	2.16
LEPVT	.069	.00067	.97	-.00063	-4.79**
LET	.082	.00067	.82	-.00063	-4.79**
LUT	.0042	.00035	7.53	-.00018	-1.04
LUR	.050	.0039	7.03	-.0017	-.84

** Significant at 5%

*** Significant at 1%

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Footnotes for Chapter 8

1. See Tella [1964, 1965], Strand and Dernberg [1964], Mincer [1965], Kuh [1966], Black and Russell [1969], and Black and Kelejian [1970]. The effect noted outweighs another response, which is that some secondary labor force members join the labor force when the primary labor force join the ranks of the unemployed. This is discussed in Bowen and Finigan [1969], which also supplies a good general reference source on this subject.
2. The presentation of equation estimates in this text is explained below equation (3.22).
3. See especially Oi [1962] and Fair [1969] on these points.
4. This function is widely employed in applied studies. We have adopted it here as the simplest equation form with acceptable economic characteristics - which a linear form does not possess. See Nerlove [1965] for an extensive analysis of the Cobb-Douglas production function.
5. The quantity t is time, and e is the base of the system of natural logarithms.
6. Black and Russell [1969, p. 72] also encountered this problem, as have other investigators.
7. See the discussion below equation (6.2); natural logarithms are used throughout this text.

8. For alternative approaches, see Kuh [1965 b], and Black and Kelejian [1970].
9. On the use of the F-test to examine linear restrictions on regressive coefficients, see Fisher [1970].
10. This may serve to illustrate why forecasting is a risky business. A forecaster inevitably makes errors; if they cancel he is a hero, while if they add he is a goat. Being right for the wrong reasons is infinitely better (outside academia) than being wrong.

CHAPTER 9

WAGES AND PRICES

In this chapter, an equation for the average compensation per man-hour in the private sector of the economy is derived. Based largely on this wage rate variable, equations for four endogenous implicit price deflators are also presented. Our approach to the wage-price sector began rather conventionally, but we have ended up adopting a somewhat novel wage equation, since the standard specifications seem to have weak theoretical and statistical foundations.

We first discuss the data series used in this sector. We next take up our model of wage determination and then consider the price equations. The chapter concludes with a discussion of simulation experiments.

9.1 THE DATA

The variables appearing in this chapter are listed in Table 9.1. The first of these, WRPVT, was computed according to

$$(9.1) \quad \text{WRPVT} = \text{YPRCE} / \text{LPEHR},$$

where YPRCE is private compensation of employees, equal to total compensation of employees from the National Income Accounts minus gross government product (compensation of government employees:

Table 9.1

Variables Appearing in Chapter 9
and in the Wage-Price Sector

(Endogenous Variables - Discussed in Chapter 9)

WRPVT	Private Sector Average Wage Rate (current dollars per hour, SA).
PGGP	Implicit Price Deflator for Gross Government Product (1957-59 = 100).
PGPP	Implicit Price Deflator for Gross Private Product (1957-59 = 100).
PCTOT	Implicit Price Deflator for Total Consumption Spending (1957-59 = 100).
PGGD	Implicit Price Deflator for Government Purchases from the Private Sector (1957-59 = 100).
LPEHRE	Equilibrium Total Man-Hours Paid For in the Private Sector (billions of hours, components SAAR).
ULC	Unit Labor Cost in the Private Sector (current dollars per constant dollar of output, SA).
ULCE	Equilibrium Unit Labor Cost in the Private Sector (current dollars per constant dollar of output, components SA).
AVP	Average Value Product of Labor in the Private Sector (current dollars per man-hour, SA).

AVPE Equilibrium Average Value Product of Labor in the
Private Sector (current dollars per man hour,
components SA).

(Endogenous Variables - Determined Elsewhere in the Model)

YGPP Real Gross Private Product (billions of constant dollars,
SAAR).

CTOT Total Consumption Expenditures (billions of constant
dollars, SAAR).

LPEHR Total Man-Hours Paid For in the Private Sector (billions
of hours, SAAR).

LWKHR Average Hours Paid For per Employee per Week in the
Private Sector (hours, SA).

(Exogenous Variables)

LEGC Government Civilian Employment (billions of persons).

LEGM Government Military Employment (billions of persons).

TIME Time Trend, Equals 1.0 in 1954I and Rises by 1.0 Each
Quarter.

SA = Seasonally-Adjusted

SAAR = Seasonally-Adjusted Quarterly
Total Measured at Annual Rates

LPEHR is an unpublished series furnished by the Bureau of Labor Statistics and described in Table 9.1.¹

The next three endogenous variables determined in this sector were taken directly from the National Income Accounts. The fourth endogenous variable, the deflator PGGD, was computed as follows. Total government purchases of goods and services minus compensation of government employees is defined as GGDI, when both quantities are in current dollars from the National Income Accounts. This difference is defined as GGD when both are the National Income Accounts' constant dollar estimates.² Then PGGD is simply one hundred times the ratio of GGDI to GGD.

The next quantity, LPEHRE was obtained by solving the private sector man-hour demand equation, (8.12), for equilibrium man-hour demand as a function of YGPP and TIME. The resultant identity, which was used to compute LPEHRE, was

$$(9.2) \quad \log(\text{LPEHRE}) = - .005885 \text{ TIME} + .8186 \log(\text{YGPP}).$$

The last four endogenous variables determined here are introduced merely to simplify presentation of equations. They are of no special interest themselves, and they are defined when used below.

The first two endogenous variables used here but determined elsewhere were taken directly from the National Income Accounts. LPEHR was described above, and LWKHR was computed according to

$$(9.3) \quad \text{LWKHR} = \text{LPEHR}/(52 \text{ LEPVT}),$$

where LEPVT is the quarterly average of seasonally-adjusted monthly estimates of total employment in the private sector taken from Employment and Earnings.³

The first two exogenous variables listed are also quarterly averages of monthly figures from the survey, and the third is self-explanatory.

9.2 WAGE DETERMINATION

The standard model of wage determination is based on the Phillips Curve, first presented in Phillips [1958], which says that the tighter are labor markets, as measured, usually, by the reciprocal of the unemployment rate, the more rapidly wages rise. This treatment has its origins in the notions that price in any market rises when there is excess demand and that the reciprocal of the unemployment rate is a good measure of excess demand in the labor market.⁴ Recent investigators have elaborated upon this formulation by allowing for lags in the rate of change of wages, the impact of consumer prices, and the effect of corporate profits, among other things: see Lipsey [1960], Perry [1966], deMenil [1969], Lucas and Rapping [1969], and deMenil and Enzler [1970].⁵

We experimented with a number of variations of this theme. All such specifications were consistently out-performed by a model due to Kuh [1967].⁶ This approach follows standard price theory closely by postulating that the equilibrium level of wages is proportional to the marginal value product of labor, which in turn varies with the average value product of labor. Taking the private sector as a whole, the average value product of labor, AVP, is defined by

$$(9.4) \quad \text{AVP} = [(\text{PGPP})(\text{YGPP})]/\text{LPEHR}$$

This approach permits the actual wage to adjust gradually to its equilibrium level. All equations employed were linear in the logarithms of the variables involved, as this specification is appropriate if the aggregate production function is of the Cobb-Douglas form discussed in Section 8.2.

Kuh found that when AVP is used in wage equations, neither measures of labor market tightness, such as the unemployment rate or its reciprocal, nor the level of corporate profits have significant coefficients. We also found this to be true. Similarly, we found, as did Kuh, that consumer prices exerted only a transient (though statistically significant) influence upon wage levels. No measure of the rate of change of YGPP nor any measure of capacity utilization, such as LWKHR or the change in LWKHR, had a significant coefficient.

Kuh's basic formulation was modified by adopting an approach often used in studies of price determination and in measurements of potential output.⁷ Since labor is not hired or fired instantaneously whenever demand rises or falls, observed productivity is affected by the fact that actual manhours paid for are rarely equal to equilibrium manhours, because observed labor inputs adjust with some lag to cyclical variation in output.⁸ Since labor and management are well aware of this fact, one might expect cyclical variations in AVP to have only a minor impact on wage determination. We consequently distinguish between the observed average value product of labor, AVP, and the normal or equilibrium average value product of labor, AVPE. We tried simple moving averages of past values of AVP as an approximation of AVPE, just as Kuh did, but this variable was consistently inferior to the following construction:

$$(9.5) \quad AVPE = (PGPP \ YGPP) / LPEHRE,$$

where LPEHRE is defined by (9.2).

The best equation found for WRPVT, and the one appearing in the model, is the following:⁹

$$\begin{aligned}
 (9.6) \quad \log(\text{WRPVT}) = & - 3.207 + .1784 [\log(\text{AVP}) - \log(\text{AVPE})] \\
 & (5.21) \quad (2.75) \\
 & + .6108 \log(\text{AVPE}) + .4110 \log[\text{WRPVT}(t-1)] \\
 & (5.22) \quad (3.62) \\
 & + .7870 \Delta \log(\text{PCTOT}) \\
 & (3.06)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .00540 \\
 DW &= 1.76
 \end{aligned}$$

The symbol Δ is the difference operator introduced in Chapter 2. Multiplying the standard error of (9.6) by the sample mean of WRPVT, we obtain an approximate standard error in terms of that quantity of 1.2¢ per hour.¹⁰

The mean lag of (9.6) is just over two months, much shorter than the lags encountered in most previous work. A variety of factors contribute to an explanation of this finding. First, WRPVT measures average compensation paid per manhour, and thus it includes overtime as well as straight-time earnings. Changes in demand and thus in AVP which lead to changes in the amount of overtime worked thus affect WRPVT immediately, even in the absence of any change in contracted wage rates. Second, much non-union labor is included in the total private sector, and their wage rates adjust more rapidly than do union wages, which are typically set by two- or three-year contracts. Finally, the major determinant of wages in this model, equilibrium labor productivity, is an extremely smooth series, so that while wage changes are

themselves smooth and small, the indicated adjustment speed is rapid.

The long-run elasticity of wages with respect to AVPE is 1.04, which is nearly equal to one, as we would expect.¹¹ Equation (9.6) clearly shows that the equilibrium value of AVP is more important than deviations from equilibrium which occur as labor input is being adjusted. Consumer prices, measured here by PCTOT, exert an important transient influence on WRPVT, but the level of PCTOT does not affect the equilibrium level of wages.

Not only is (9.6) more in line with neo-classical price theory than the usual Phillips Curve approach, but it is statistically more powerful as well. Our model, in contrast to the more common formulation, implies that unemployment has negligible if any influence on wages. We have been unable to find any convincing evidence for such an influence in our data. The much larger and more competitive non-union sector is the readiest explanation for this heterodox finding.

Perhaps the Phillips Curve doctrine is more applicable to periods of considerable unemployment or when unemployment fluctuates more than in our sample, where the range is 3.4% - 7.4%. The rather neoclassical approach of this paper is perhaps most consistent with a situation of full employment, a condition more nearly met since World War II than at any other time in our history.

9.3 PRICE EQUATIONS

Price of Gross Government Product

The implicit deflator for gross government product, PGGP, is conceptually simple; it is just a weighted average of government wage rates. These should be related to the average wage rate paid in the private sector, possibly with a lag. If there has been a difference in the rate of growth of civilian and military wages, the civilian-military mix of government employment should affect the relation between the average government wage and the average private wage.

The best equation found for PGGP is the following:

$$\begin{aligned}
 (9.7) \text{ PGGP} = & 2.607 + 13.88 \text{ WRPVT} - 2.913 [\text{WRPVT LEGM}/(\text{LEGM}+\text{LEGC})] \\
 & (1.84) \quad (3.70) \quad (1.25) \\
 & + .7527 \text{ PGGP}(t-1) \\
 & (10.7)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .999 \\
 SE &= .7946 \\
 DW &= 2.09
 \end{aligned}$$

In spite of the high R^2 , this is not a particularly strong equation, as the lagged dependent variable provides most of the explanatory power. The mean lag is 3.1 quarters, which seems reasonable. We retain the third term even though it is not significant at the 10% level because it correctly indicates the effect of military wages rising less rapidly than the wages paid civilian government employees.

Price of Gross Private Product

Our equation for the implicit deflator for gross private product, PGPP, is based closely on the work of Schultze and Tyron [1965] and Eckstein and Fromm [1968].¹² We assume that prices are basically determined as a markup on unit labor cost. This quantity is defined for the entire private sector by

$$(9.8) \quad \text{ULC} = (\text{WRPVT LPEHR})/\text{YGPP}$$

Following the studies just cited, we distinguish between observed unit labor cost, ULC, and normal or equilibrium unit labor cost, ULCE. We experimented with use of moving averages of past values of ULC as a proxy for ULCE, but we obtained better results using equilibrium manhours much as we did in the wage equation and defining

$$(9.9) \quad \text{ULCE} = (\text{WRPVT LPEHRE})/\text{YGPP},$$

where LPEHRE is defined by equation (9.2).

Presumably prices adjust to costs with some sort of a lag, so all estimates allowed for Koyck lags in price determination. Further, the price level ought to depend on how "tight" product markets are, that is, on short-run pressures on currently-employed labor and capital. As a measure of these pressures, we took the ratio of LWKHR, average weekly hours paid for in the private sector, to its equilibrium value.¹³ We assumed equilibrium

weekly hours to be a decaying exponential function of time over the sample period, an assumption consistent with the observed secular decline in average hours worked, which is due in large measure to an increasing proportion of part-time female employees in the labor force. Thus, in an equation linear in the logarithms, both $\log(\text{LWKHR})$ and TIME would be expected to have positive coefficients.

The equation for PGPP used in the model is consistent with this discussion and with earlier investigations:

$$\begin{aligned}
 (9.10) \quad \log(\text{PGPP}) = & 1.472 + .1537 [\log(\text{ULC}) - \log(\text{ULCE})] \\
 & (3.91) \quad (5.11) \\
 & + .2776 \log(\text{ULCE}) + .6489 \log[\text{PGPP}(t-1)] \\
 & (7.49) \quad (11.7) \\
 & + .08461 \log(\text{LWKHR}) + .0003161 \text{ TIME} \\
 & (2.28) \quad (3.14) \\
 & R^2 = .9995 \\
 & \text{SE} = .00198 \\
 & \text{DW} = 1.98
 \end{aligned}$$

Multiplication of the standard error of this equation by the sample mean of PGPP yields an approximate standard error of this variable of .21.

This equation has a mean lag of 1.85 quarters or 5.5 months, considerably longer than the mean lag of the wage equation. As most studies of price formation have found, we note that ULCE is more important than short-run deviations of observed ULC from

this equilibrium value. The long-run elasticity of price with respect to unit labor cost is .79, which is close to but not equal to unity, the value we would expect to find if equilibrium markup of price over unit labor cost were constant.¹⁴ Our estimate implies that in equilibrium prices do not rise quite as fast as unit labor costs. This is a surprising finding: the only explanations that come to mind are changes in the intensity of competition or in the importance of various sectors of the economy during the sample period.

The steady-state properties of the wage-price system are worth some comment at this point. If we let AP be the average (real) product of labor, $YGPP/LPEHR$, and let AP^* be the equilibrium value of this quantity, equation (9.6) determines the steady-state value of the money wage, $WRPVT$, which we can call W^* , as basically a constant times the product of AP^* and P^* , the steady-state value of $PGPP$, the price level. Similarly, equation (9.10) determines P^* as a constant times $[W^*/AP^*]$.⁸ Essentially, then, these two equations make the equilibrium real wage, W^*/P^* , a function of steady-state labor productivity, AP^* . The equilibrium absolute price and money wage levels are mainly determined elsewhere in the system; the money supply is very important here, as we shall see in Chapter 10.

This is basically a Keynesian-Classical full employment model, which is not a priori unsuited to the post-war U.S. economy and

seems more consistent with the data than most alternative versions we have seen. Models are, after all historical representations: it would be most remarkable for this or any alternative model to capture the essence of any economy in all extreme conditions, however attractive this would be. The use of this model to simulate extreme situations - at least so far as price/wage behavior is concerned - should be undertaken with the most extreme caution.

Other Deflators

We now must determine the price deflators for total consumption and for government goods and services purchased from the private sector, PCTOT and PGGD. As we have no industry data in this model, it is impossible to relate these prices to their fundamental determinants as we did for PGPP. Instead, we assume that the rate of change of these deflators is straight-forwardly related to the rate of change of the deflator for gross private product. Also, it is reasonable to assume that the more important these categories are in the total demand for private output, the more rapidly their prices will rise. A shift of demand to consumer goods, for instance, will generally force the prices of such goods up more rapidly than the average of private sector prices.

We used the change in the logarithm as an approximation to the percentage change of the various deflators, a valid approximation

when the percentage changes are small.¹⁵ A variety of specifications were tried, most notably some involving the change in the logarithm of WRPVT. All of the latter failed.

Price of Consumption Outlays The best equation for the implicit deflator for total consumption, PCTOT, was

$$\begin{aligned}
 (9.11) \quad \Delta \log(\text{PCTOT}) = & \quad -.04468 + .7902 \Delta \log(\text{PGPP}) \\
 & \quad (2.29) \quad (11.8) \\
 & \quad + .1518 \Delta \log(\text{PGPP}(t-1)) + .06340 (\text{CTOT}/\text{YGPP}) \\
 & \quad (2.45) \quad (2.29) \\
 & \quad R^2 = .782 \\
 & \quad \text{SE} = .00168 \\
 & \quad \text{DW} = 1.85
 \end{aligned}$$

Multiplying the standard error of (9.11) by the sample mean of PCTOT yields an approximate standard error of .176. Notice that even though the standard error of (9.11) is somewhat smaller than that of (9.10), the R^2 is dramatically lower. Published price indices are largely trend, so it is not too difficult to obtain good fits to the level of prices. Explaining the rate of change of prices, however, is far more difficult.

All coefficients in (9.11) are significant with the expected signs, and the overall fit is acceptable. The sum of the coefficients of current and lagged percentage changes in PGPP is .94, so that PCTOT is almost exactly proportional to PGPP in equilibrium.

Price of Government Purchases from the Private Sector A

large number of equations, most of which were variants of the specification embodied in (9.11), were tried for this variable. Results were uniformly disappointing. Few terms were significant, and those that were generally had perverse signs. We finally settled on the following exceedingly crude equation:

$$(9.12) \quad \Delta \log(\text{PGGD}) = 1.319 \Delta \log(\text{PGPP})$$

(8.42)

$$R^2 = .330$$

$$SE = .00819$$

$$DW = 2.16$$

The approximate standard error of PGGD is .854, much larger than that of PCTOT.

The Durbin-Watson statistic, surprisingly in view of the poor fit and extremely simple specification, does not suggest the omission of any systematic influences on PGGD. Cynical readers of Joint Economic Committee reports on government procurement practices may be able to rationalize the large coefficient of the price change variable. This term is significantly different from unity at the 5% level.

9.4 SIMULATION RESULTS

Dynamic simulations were performed on the wage-price sector, consisting of behavioral equations (9.6), (9.7), and (9.10)--(9.12),

along with identities (9.2), (9.4), (9.5), (9.8), and (9.9).¹⁶

The first of these identities determines LPEHRE as a function of quantities exogenous to this sector, and the last four compute quantities of no great intrinsic interest. Consequently, simulation results are presented only for WRPVT and the four implicit price deflators determined by this sector.

Sample period simulation results, reported in Table 9.2, are quite acceptable. The two key variables, WRPVT and PGPP, are forecast with small errors and no bias. Given good forecasts of the private wage, the PGGP equation responds with good estimates of that deflator. The significant upward bias in PCTOT is mildly disturbing, especially since PGPP is, on average, slightly underestimated, and it shows that this equation has a tendency to go off the track in long simulations. PGGD is, as we would have expected, not very well explained in this run; the RMS error is large, and there is evidence of bias, though some of the latter problem is due to the small downward bias in PGPP.

Table 9.3 indicates that we obtained considerably less encouraging results for the four quarters of 1970. All RMS errors are considerably larger than their counterparts in Table 9.2. Further, the t-tests here understate considerably the failure of this sector.

All errors for all variables were negative, and all became more negative uniformly over time.¹⁷ The estimate of WRPVT in

Table 9.2

Wage-Price Sector Simulation Results:1954I - 1969IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
WRPVT	2.15	.039	1.58	-.0053	-1.09
PGPP	104.90	1.22	1.13	-.181	-1.19
PGGP	118.10	1.82	1.40	-.024	-.10
PCTOT	105.05	1.39	1.26	.511	3.13**
PGGD	104.26	3.34	3.34	-.759	-1.85*

* Significant at 10%

** Significant at 5%

Table 9.3

Wage-Price Sector Simulation Results:1970I - 1970IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
WRPVT	3.57	.281	7.53	-.230	-2.47*
PGPP	130.13	4.34	3.28	-3.36	-2.12
PGGP	186.65	4.66	2.49	-4.62	-13.48***
PCTOT	129.25	3.48	2.65	-2.66	-2.06
PGGD	132.30	6.06	4.49	-4.67	-2.10

* Significant at 10%

*** Significant at 1%

1970IV was approximately equal to the 1970I estimate, while the estimates of PGPP showed a slight decline over the year, even though both series actually increased considerably in 1970. In short, the model did not even begin to capture the inflation that occurred in 1970. We note defensively that many forecasters at the time made the same sort of mistake and that there still is no consensus within the economics profession about the driving forces behind the inflation of the last few years.

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Footnotes for Chapter 9

1. See Section 3.1 for the procedures employed in taking series from the National Income Accounts. YPRCE appears explicitly in Chapter 7.
2. GGDI and GGD appear explicitly in Chapter 11, and the latter is also present in the model-closing identities presented in Chapter 12.
3. LEPVT appears explicitly in Chapter 8.
4. Of course this approach does not take into account another, equally plausible but harder to measure, indicator of excess demand: the ratio of job vacancies to employment (or unemployment). A number of interesting discussions of the micro-theoretic foundations of the Phillips relation may be found in Phelps, et al [1970]; see also Samuelson and Solow [1960].
5. The papers by Lucas and Rapping [1969], which attempts to take account of inflationary expectations, and by de Menil and Enzler [1970] are especially interesting. In the latter, the labor market is divided into unionized and non-unionized sectors, and contract negotiations, minimum wages, composition effects, and overtime hours, among other refinements, are explicitly treated. Within our mandate of relative simplicity of form, variables, and exposition, it is not possible to

incorporate such quantities. How much we have thus sacrificed remains to be seen.

6. Sargan [1964] has proposed a similar model. See also Black and Kelejian [1970]. When quite intricate Phillips Curves, including many plausible subsidiary variables, were estimated for the M.I.T. - F.R.B. - Penn model, apparently superior statistical estimates were obtained; see de Menil and Enzler [1970]. But the key difference was the use of overlapping two-quarter differences for the dependent variable instead of the usual one quarter (independent) changes. In our view, the statistical status of this transformation is too shaky to justify its use.
7. See Schultze and Tyron [1965], Eckstein and Fromm [1968], and Black and Russell [1969].
8. This point is discussed at some length (and a number of references are cited) in Section 8.2.
9. The presentation of estimated equations in this text is discussed immediately below equation (3.22); natural logarithms are used throughout this text.
10. See the discussion below equation (6.2).
11. If this elasticity were exactly unity, the implication would be that the share of wages in National Income would be constant, an implication which also follows from an assumption that the aggregate production function is Cobb-Douglas.

(See Section 8.2.) Our estimate of just above unity implies a slowly increasing wage share, which conforms more closely to the historical facts. Kuh [1967] obtained similar estimates of this elasticity, though his equations showed much longer lags than ours.

12. For a rather different approach, aimed specifically at developing an equation for forecasting price movements, see Steckler [1968].
13. Overtime and short time are the most flexible controls available to business managements faced with the need to adjust manhour inputs to short-run output changes. See Holt, et al [1960], Kuh [1965], and Section 8.2 above. Thus changes in hours worked per employee per week are very closely related to short-run variations in capacity utilization and demand pressure.
14. For those who are familiar with the concept, the asymptotic standard error of this long-run elasticity is about .05, which signals that it is significantly different from unity.
15. See the discussion below equation (6.2).
16. Other interesting simultaneous equation wage-price models to which the reader might refer are Dicks-Mireaux [1961] and Sargan [1964].

17. A weakness in the t -test for bias, illustrated here, is that when there is considerable variance in the errors, even though they may be all of the same sign, it is hard to reject the hypothesis that the mean error is zero using this test.

CHAPTER 10
FINANCIAL MARKETS AND MONETARY POLICY

Increasing attention has been paid, both in academic research and in government, to the interaction of financial markets with the real economy. As a reflection of this our model has a more elaborate financial sector than has been traditional for models of its size. The main task of this sector is to determine the corporate bond, treasury bill, and commercial paper rates. These influence the fixed investment and income distribution sectors.

Most serious work on monetary econometrics is quite recent. This sector relies heavily on the work of Frank de Leeuw [1965, 1969], Patric H. Hendershott [1968], and a group at M.I.T., the Federal Reserve, and the University of Pennsylvania, who have constructed an elaborate model of the U.S. economy. We have drawn on an early version of the MIT-FRB-Penn financial sector presented by de Leeuw and Gramlich [1968]. Financial markets are complex and subject to frequent structural changes, thereby making it exceptionally difficult to model their basic structure. We have less confidence in the validity of most equations reported here than those appearing in most other chapters.

We first discuss the data series employed in this sector and the relation between our model and monetary policy as implemented by the Federal Reserve. The next two sections discuss equations and simulation results, while policy implications are deferred to

Chapter 12 where full model simulations appear.

10.1 DATA AND INSTRUMENTS OF MONETARY POLICY

The data series employed in this chapter are listed in Table 10.1. RESR and RESF are two-month averages, centered around the end of the quarter, of daily figures from the Federal Reserve Bulletin. Since both these variables are stocks, one can expect forces which operate during each quarter to affect their levels at the end of each quarter. Hence we followed the M.I.T. - F.R.B. - Penn model and used averages for financial stocks centered on the end, rather than the middle, of each quarter.

RESR was seasonally adjusted using the X-11 variant of the Census Bureau's Method II for seasonal adjustment.¹ The exogenous quantity RESU was similarly adjusted, and the RESF series employed here was calculated from equation (10.2), presented in the next section.

The next three quantities are the simple averages of monthly figures from the Federal Reserve Bulletin. The commercial loan rate, RCL, is the average bank rate on short term commercial and industrial loans. It was originally based on data for nineteen large cities, but in recent years the sample has been expanded to encompass thirty-five cities.

There are no quarterly figures for the time deposit yields which are publicly available, though an annual series can be computed from data in the annual reports of the Federal Deposit Insurance

Table 10.1
Variables Appearing in Chapter 10
and in the Financial Sector

(Endogenous Variables - Discussed in Chapter 10)

RESR	Required Reserves of Federal Reserve Member Banks (billions of current dollars, SA).
RESF	Free Reserves of Federal Reserve Member Banks (billions of current dollars, SA).
RCB	Average Yield on Aaa-Rated Corporate Bonds (percent per year).
RCP	Average Yield on 4-6 Month Prime Commercial Paper (percent per year).
RTB	Average Yield on 3 Month U.S. Treasury Bills (percent per year).
RCL	Average Bank Rate on Short-Term Commercial and Industrial Loans (percent per year).
RTD	Interest Rate on Time Deposits at Commercial Banks (percent per year).
MTD	Time Deposits at All Commercial Banks (billions of current dollars, SA).
MDDP	Net Demand Deposits (Adjusted) Other than U.S. Government at All Commercial Banks (billions of current dollars, SA).
MDD	Total Net Demand Deposits (Adjusted) at All Commercial Banks (billions of current dollars, SA).
MCL	Commercial and Industrial Loans at All Commercial Banks (billions of current dollars, SA).

(Endogenous Variables - Determined Elsewhere in the Model)

YGNPI	Gross National Product (billions of current dollars, SAAR).
YGPPI	Gross Private Product (billions of current dollars, SAAR).

Table 10.1
Variables Appearing in Chapter 10
and in the Financial Sector

Continued

(Exogenous Variables)

RRQD	Implicit Weighted Average of Required Reserve Ratios Against Demand Deposits of Federal Reserve Member Banks (fraction).
RRQT	Reserve Requirement Ratio Against Time Deposits of Federal Reserve Member Banks (fraction).
RTDMX	Legal Maximum Rate Payable on Commercial Bank Time Deposits (percent per year).
DR	Federal Reserve Bank of New York Discount Rate (percent per year).
RESU	Unborrowed Reserves of Federal Reserve Member Banks (billions of current dollars, SA).
MDDFG	Federal Government Demand Deposits at All Commercial Banks (billions of current dollars, SA).
DQ	Dummy Variable for 1969 and 1970, Equal to 1.0 Then, Zero Otherwise.
DUCD	Dummy Variable for the Creation of Certificates of Deposit, Equal to Zero Before 1962 I, One Thereafter.

Corporation. We employed an unpublished series for RTD, supplied us from the data bank of the M.I.T. - F.R.B. - Penn model and obtained by those investigators from the Chicago Federal Reserve District. The Chicago Federal Reserve District takes a survey of advertised passbook and consumer C.D. rates. These are averaged within each of the two categories and weighted by fund inflows, rather than totals, in order to emphasize marginal rate behavior.

MTD and MDDP are two-month seasonally-adjusted averages, centered on the end of the quarter, of daily figures from the Federal Reserve Bulletin. The gross demand deposit variable, MDD, was computed from the identity

$$(10.1) \quad MDD = MDDP + MDDFG,$$

where MDDFG is an exogenous variable discussed below. The commercial and industrial loan series, MCL, is an unpublished series prepared and seasonally adjusted by the Board of Governors of the Federal Reserve System, at all commercial banks.

The two endogenous variables determined elsewhere in the model were taken directly from the National Income Accounts, as outlined in Section 3.1.

Required reserves for time deposits, RRQT, was obtained from the M.I.T. - F.R.B. - Penn model data bank by multiplying the official reserve requirement times total time deposits at member banks. RRQT is then subtracted from RESR (obtained from the identity (10.2) below) and divided by demand deposits subject to reserve

requirements to yield RRQD, required reserves for demand deposits. RTDMX, the maximum time deposit rate, is obtained from the same source as RTD, an unpublished survey by the Chicago Federal Reserve District, and weighted by fund flows in the same way.

MDDFG is a two-month average of daily figures from the Federal Reserve Bulletin, centered on the end of the quarter. The first dummy variable, DQ, is introduced to account for the heavy activity in the commercial paper market associated with the one-bank holding companies in the 1969-70 period. The second dummy variable, DUCD, reflects the changes in the structure of financial markets which occurred with the introduction of certificates of deposit.

The Federal Reserve has four basic types of policy tools at its disposal. First, it can fix some of the interest rates on which banks and individuals base their decisions. For instance, the rate of interest on checking accounts is fixed (by law) at zero. The two principal non-zero interest rates determined by the Federal Reserve are present in this sector; they are DR and RTDMX. It should be noted, though, that both these rates are adjusted by the Federal Reserve passively as well as actively. Both rates are often moved to keep them "in line" with other interest rates, rather than to cause changes in financial market conditions. Since it is not at all clear how to separate movements of these rates into endogenous and exogenous components, we follow all other authors and treat these two rates as exogenous.

The second set of tools the Federal Reserve can use is the reserve requirements ratios of member banks. Different classes of banks have different required reserve ratios, but it would unduly complicate our model to take this into account. Since the fractions of deposits in each class of bank are nearly constant, we lose almost no realism by summarizing the reserve requirements policy parameters as RRQD and RRQT.

The third tool of the Federal Reserve is more frequently utilized than the first two. The Federal Reserve buys and sells government securities in the open market. A purchase of securities adds to the unborrowed reserves of member banks, while a sale reduces reserves. Such purchases and sales, called "open market operations", are undertaken on a daily basis. We represent this tool in our model by the policy variable RESU.

It should be noted that while RESU is treated as exogenous, what the Federal Reserve Systems controls to the last dollar is its own portfolio of government securities, not unborrowed reserves. The difference between unborrowed reserves and Federal Reserve holdings of government securities consists of the gold stock, currency, and minor items such as Treasury and cash accounts and float, which sometimes fluctuate sharply from week to week. Treating RESU as a policy variable amounts to assuming that the Fed can, and generally does, offset quarterly movements in the gold stock and currency.

Finally, the Federal Reserve can limit member bank borrowings.

The Fed is not required to lend reserves to banks that seek to borrow them, and one often hears that the Fed "frowns" at borrowing or exercises "moral suasion". However, in the short run banks can borrow, but at increasing costs. One popular text puts the matter as follows:²

A Reserve bank rarely refuses to lend to a member bank that is facing an actual or prospective deficiency in its reserves. But after making a short-term loan to a bank, it studies the situation carefully. If it finds that the bank has borrowed too often, too continuously, too much, or for improper reasons, it may advise the bank to contract its loans or sell securities in order to reduce or retire its borrowings. It may even go as far as to refuse to renew the loan, and in extreme cases it may suspend the bank's borrowing privilege.

Federal Reserve officials could, of course, attempt to regulate the volume of member-bank borrowing by varying their own attitudes toward lending, being very strict at some times and more liberal at others. Though this is done to some extent, it is not a very flexible or effective instrument.

The financial sector thus contains five monetary policy instruments under the control of the Federal Reserve: DR, RTDMX, RRQT, RRQD, and RESU. We could carry out policy simulations of this sector in isolation, not allowing for feedbacks from the rest of the economy.³ It seems more sensible, however, to examine the impact of monetary

policy on the economy as a whole in Chapter 12.

10.2 THE EQUATIONS

Monetary theory typically divides the economy into three sectors: commercial banks, the non-bank public, and the government (which includes the Federal Reserve System). Based on market interest rates, requirements for transactions, current and expected income, wealth, and tastes, the non-bank public decides in what form to hold its wealth. A decrease in one interest will lead to a shift away from that asset into others which yield a higher return. The private demand for money depends primarily on transactions demand and transaction costs. This sector also decides on the level of bank loans it desires. Commercial banks decide at what terms they will accept time deposits and issue commercial loans as well as the amount of government securities they are willing to hold. The government, as mentioned above, makes reserves available to the banking system, fixes reserve requirements for Federal Reserve member banks, and sets certain interest rates.

The non-bank public can choose from among a variety of financial assets as stores of wealth. Besides demand and time deposits, there are common and preferred stocks, a variety of corporate and government bonds, commercial paper, and savings and loan shares. (Non-bank financial intermediaries such as savings and loan associations and mutual savings banks should be distinguished from the rest of the non-bank public in a less aggregative treatment.)

A rigorously correct analysis must consider all assets and liabilities for one simple reason. For any economic agent or sector, assets always equal liabilities plus net worth. Failure to consider this budget constraint explicitly can lead to rather fundamental mis-specification. Brainard and Tobin [1968] and Tobin [1969] have stressed this point most convincingly. Yet we do not incorporate the sectoral budget constraints into our model.

We offer two reasons for this omission. First, a model considering all financial assets and liabilities and all corresponding interest rates would be excessively complicated for the objectives of this text. The evidence suggests that unless sophisticated estimation methods are employed, such an approach will not yield satisfactory statistical results.⁴ Second, our aim throughout this text is to describe the state of the art of macroeconometric model building. With due possible exception of the M.I.T. - F.R.B. model, no comprehensive financial models exist (as far as we know) which adequately consider sectoral budget constraints.

This sector focuses on a limited number of structural relationships, with heavy emphasis on bank behavior. Our first equation is the identity relating the three categories of bank reserves listed in Table 10.1:

$$(10.2) \quad \text{RESF} = \text{RESU} - \text{RESR}.$$

The other equivalent way to define free reserves is as the difference between banks' excess reserves and reserves borrowed from the Fed.

The first stochastic equation relates RESR to the level of commercial bank deposits.

Eight behavioral equations comprise the remainder of the sector. We estimate supply and demand equations for commercial loans and time deposits and a demand equation for demand deposits. The supply equation for this asset is implicit in the portfolio behavior of banks represented by the demand for free reserves equation (10.9). Equations relating the commercial paper and corporate bond rates and the commercial paper and treasury bill rates round out the sector.

The eleven equations of this sector determine the eleven endogenous variables listed in Table 10.1. This sector differs from the others presented since it is not the case that each endogenous variable is the dependent variable in one structural equation. Free reserves appear on the left of two equations, while RTB appears only as an independent variable. To employ these equations in simulation, we had to re-normalize some of them by rewriting at least one equation so that RESF and RTB are the dependent variables of only one equation each.

Required Reserves

If MDDP, MDDFG, and MTD were deposits only at Federal Reserve member banks, and if RESR did not include required reserves held against inter-bank deposits, we would have the following identity:

$$\text{RESR} = \text{RRQD} (\text{MDDP} + \text{MDDFG}) + \text{RRQT MTD}.$$

But none of the if's in the last sentence hold. Not all deposits are held with Federal Reserve member banks, though the fraction held there is reasonably stable. In addition, some required reserves are held against inter-bank deposits, which are netted out of MDDP. To approximate these influences on total reserves, the following equation was estimated based on the work of de Leeuw and Gramlich [1968], and de Leeuw [1969]:⁵

$$(10.3) \quad \text{RESR} = 2.417 + .7804 (\text{RRQD MDD}) + .6488 (\text{RRQT MTD})$$

(7.06)
(34.4)
(27.2)

$$\begin{aligned} R^2 &= .996 \\ SE &= .174 \\ DW &= 1.07 \end{aligned}$$

The R^2 indicates that this simple approximation does a good job of explaining movements in required reserves, but the low Durbin-Watson emphasizes that in constructing an approximation, we have mis-specified the relation. The second and third coefficients in (10.3) indicate that about 78% of demand deposits and 65% of time deposits are held with Federal Reserve member banks.

Commercial Loans

In the short run, it appears reasonable to model banks as passive responders to the demand for commercial loans; see Modigliani, Rasche, and Cooper [1970] on this point. That is, banks are assumed to attempt to satisfy all quality demand for commercial loans, because loan demanders are typically valued customers. In the short run, banks react to changes in loan demand by altering

their holdings of other assets. To some extent the commercial loan rate is changed as a means of bringing loan demand into line with banks' desired holdings of loans, but this process operates with a sizeable lag.

The demand function for commercial loans is in the spirit of the work of Goldfeld [1969]. The higher the loan rate relative to other interest rates, the lower one would expect loan demand to be. Similarly, the higher the level of real economic activity and the higher the price level, the greater should be business's need for working capital. These latter two influences can be summarized in current dollar gross private product, YGPPI.

Considerable experimentation was required to produce a sensible commercial loan demand equation. Specifications that involved scaling the dollar variables by YGPPI, as described below, failed rather miserably, as did log-linear equations. Dividend payments and inventory investment, which should affect the demand for working capital, did not enter with significant coefficients. The best equation obtained was the following:

$$(10.4) \quad \text{MCL} = -2.267 - .7020 (\text{RCL-RTB}) + .01910 \text{ YGPPI} + .8972 \text{ MCL}(t-1)$$

(1.75)
(4.11)
(3.48)
(25.7)

$$\begin{aligned} R^2 &= .999 \\ SE &= .636 \\ DW &= 1.54 \end{aligned}$$

This equation clearly exhibits the impacts of interest rates and economic activity on loan demand. The mean lag of 8.7 quarters,

is excessive, as is the statistical importance of the lagged dependent variable. Equation (10.4) fits well, but it is clearly quite imperfect.

The commercial loan rate is fairly sticky, but it eventually responds to movements in other interest rates. The supply equation for commercial loans used in this model is the following:⁶

$$\begin{aligned}
 (10.5) \quad \log(RCL) = & .2231 + .08316 \log(RCP) + .3170 \log(RCB) \\
 & (5.24) \quad (4.21) \quad (5.07) \\
 & + .5112 \log[RCL(t-1)] \\
 & (7.46)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .983 \\
 SE &= .0273 \\
 DW &= 1.24
 \end{aligned}$$

The Durbin-Watson suggests the presence of serial correlation, but the other statistics are quite acceptable, even though t-statistics are biased upward. The sluggishness of RCL does not appear in the mean lag of one quarter, but rather in the fact that the slow-moving long rate, RCB, is much more important here than the more volatile short rate, RCP. Multiplication of the standard error of (10.5) by the sample mean of RCL yields an approximate standard error of about .14 percentage points for that quantity.⁷

Time Deposits

Again we view banks as passive agents. They are assumed to set the time deposit rate but then to accept all deposits and to permit all withdrawals made at that rate.

The public's demand for time deposits should be affected positively by the yield on time deposits and negatively by the yield

on other assets. Higher income should increase the demand for time deposits. As usual, we allowed for gradual adjustment to equilibrium by considering a first-order lag mechanism. We have followed de Leeuw [1965] here in scaling all dollar variables by dividing them by current dollar gross national product. This forces the income elasticity of demand to unity and removes the effects of the changing scale of the economy. The equation form finally adopted is standard:⁸

$$\begin{aligned}
 (10.6) \quad \text{MTD/YGNPI} = & .00680 + .00430 \text{ RTD} - .00172 (\text{RTB} + \text{RCB}) \\
 & (4.57) \quad (5.88) \quad (9.67) \\
 & - .1963 \Delta \text{YGNPI/YGNPI}(t-1) + .9878 [\text{MTD}(t-1)/\text{YGNPI}(t-1)] \\
 & (7.75) \quad (47.3) \\
 & R^2 = .998 \\
 & \text{SE} = .00183 \\
 & \text{DW} = 1.35
 \end{aligned}$$

The quantity Δ is the difference operator introduced in Chapter 2.

Equation (10.6) fits the data quite well. Multiplication of the standard error of this equation by the sample mean of YGNPI yields an approximate standard error of MTD of \$1.07 billion, about one percent of the sample mean of this quantity. Both interest rate coefficients have the expected sign and are highly significant. The second to last term in (10.6) indicates that when income increases, time deposits are temporarily drawn down, to pay for increased durables consumption and business outlays for inventories and to meet other short-term financial needs.

The specification is clearly not perfect, however, as the low

Durbin-Watson indicates. Further, while it seems plausible that time deposit holdings move slowly to their equilibrium level, the mean lag of 81 quarters in (10.6) is excessive.

The supply function for time deposits determines RTD , the average rate at which banks are willing to accept such deposits. The equilibrium time deposit rate should be positively related to other interest rates. As banks do not like to make frequent sizeable changes in the time deposit rate, though, RTD should be affected by other rates through a distributed lag mechanism. In contrast, banks should adjust the time deposit rate quite rapidly in response to changes in the effective ceiling rate, $RTDMX$. A decrease in the ceiling rate forces some banks to lower their rates immediately, while an increase leads banks that would have had higher rates in the absence of the ceiling to raise their rates quickly. The specification adopted here incorporates both these notions, though in a rather ad hoc fashion:⁹

$$(10.7) \quad \log[RTD/RTDMX] = .03550 \log[(RCB+DR/(2 RTDMX)] \\ (2.97) \\ + .9614 \log[RTD(t-1)/RTDMX(t-1)] \\ (133.)$$

$$R^2 = .993 \\ SE = .0168 \\ DW = 2.51$$

The insignificant constant term was deleted. The equation fits well; the approximate standard error of RTD is about .051 percentage points. The mean lag is unreasonable, however, and the lagged

dependent variable is clearly providing almost all the equation's explanatory power.

Demand Deposits

In the standard Keynesian analysis, the public's holdings of demand deposits are assumed to depend positively on income and negatively on interest rates. Current dollar income reflects the need for means of payment, while interest rates measure the opportunity cost of holding wealth in non-earning form.¹⁰ Since private money holdings cannot be expected to adjust instantaneously to changes in their determinants, we allow for the operation of a first-order distributed lag. Following de Leeuw [1965, 1969] as in the time deposit demand equation and deflating by current dollar income, we obtain our demand deposit demand equation:

$$\begin{aligned}
 (10.8) \quad \text{MDDP/YGNPI} &= .0550 - .00421 \text{ RTD} - .00132 \text{ RTB} \\
 &\quad (2.97) \quad (2.38) \quad (3.55) \\
 &+ .8191 [\text{MDDP}(t-1)/\text{YGNPI}(t-1)] \\
 &\quad (14.1)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .996 \\
 SE &= .00224 \\
 DW &= 1.17
 \end{aligned}$$

As with most of the other equations in this chapter, the fit is excellent but the Durbin-Watson statistic is suspiciously low. The two interest rates employed, RTD and RTB, both relate to low risk earning assets which are highly liquid and are thus close substitutes for demand deposits. This equation has a mean lag of 4.5 quarters, which appears reasonable.

Free Reserves

Free reserves, even seasonally adjusted, are an extremely volatile series. We do not explain them particularly well, nor do other investigators. Desired free reserves, which can be thought of as a measure of banks' desired safety margins, should depend positively on the discount rate, which represents the explicit money cost of borrowing reserves should the need arise. Similarly, the higher are market interest rates, which represent the opportunity cost of holding funds in non-earning form, the lower should be desired interest rates. Actual free reserves should adjust gradually to the equilibrium level.

To eliminate the effects of changes in the scale of the banking system, dollar variables were scaled by dividing by lagged demand deposit liabilities. Finally, changes in unborrowed reserves and commercial loans, both essentially exogenous to the banking system in the short run, will cause temporary changes in the level of free reserves while other components of banks' balance sheets are being adjusted. Our free reserve equation, like most others in this chapter, is basically a standard specification:¹¹

$$\begin{aligned}
 (10.9) \quad \text{RESF/MDD}(t-1) = & .0003648 + .00200 (\text{DR} - \text{RTB}) \\
 & (1.41) \quad (4.92) \\
 & + .6633 [\text{RESF}(t-1)/\text{MDD}(t-1)] - .6277 [\text{RRQD } \Delta \text{MCL}/\text{MDD}(t-1)] \\
 & (11.4) \quad (4.34) \\
 & + .4472 [(1.-\text{RRQD}) \Delta \text{RESU}/\text{MDD}(t-1)] \\
 & (5.46)
 \end{aligned}$$

$$\begin{aligned} R^2 &= .894 \\ SE &= .00109 \\ DW &= 2.13 \end{aligned}$$

This equation fits as well as can be expected, and the individual coefficient estimates are quite respectable. The standard error in terms of RESF is about \$140 million, while the sample mean of this quantity is -\$3 million, indicating that large percentage errors in RESF are to be expected in simulation runs. The mean lag of this equation is about two quarters, implying rather rapid adjustment on the part of the banking system.

The last two terms in (10.9) require some explanation. An autonomous increase in the banking system's outstanding loans of one dollar leads to a one dollar increase in demand deposits immediately. This increase, in turn, raises required reserves by RRQD and hence lowers RESF by a like amount if no offsetting changes are made by individual banks. The second to last coefficient in (10.9) indicates, then, that about 37% of such autonomous (and, generally, undesired) changes in free reserves are offset. Similarly, if the Federal Reserve increases unborrowed reserves by one dollar through open market operations, the banking system's demand deposits also increase by a dollar in the very short run, and required reserves rise by RRQD. Thus, ceteris paribus, free reserves would rise by $(1 - \text{RRQD})$. The last coefficient in (10.9) indicates that about 55% of these changes are offset.¹²

The Structure of Interest Rates

We first consider the equation relating the corporate bond rate to the commercial paper rate. It is well known that in perfect capital markets the long-term rate of interest will always equal the average of future expected short-term rates.¹³ This is because borrowers can either borrow long or borrow short and repeatedly re-finance, and lenders can either buy long-term bonds or a sequence of short-term bonds. In reality, borrowers would rather not have to worry about refinancing, since future short rates are uncertain, and they are thus willing to pay a premium in order to sell long-term bonds. There is, therefore, a tendency for the long rate to be consistently above the short rate, a phenomenon called "normal backwardation" by J.M. Keynes.¹⁴

We assume here that expected future short-term commercial paper rates are a distributed lag function of past observed values of RCP.¹⁵ Further, we assume that RCB is the long rate corresponding to RCP. A variety of distributed lag equations were estimated; the best on statistical grounds was quite simple:

$$(10.10) \quad \log(RCB) = \underset{(2.59)}{.04275} \log(RCP) + \underset{(63.4)}{.9706} \log[RCB(t-1)]$$

$$R^2 = .981$$

$$SE = .0321$$

$$DW = 1.39$$

This equation fits fairly well; the approximate standard error in terms of rCB is about .14 percentage points. The low Durbin-Watson, the extreme importance of the lagged dependent variable, and the implausible mean lag (45 quarters) indicate, however, that it is far from perfect. The equilibrium relation

estimated between RCB and RCP is approximately

$$RCB = RCP.$$

The final equation in the financial sector relates the commercial paper rate to the treasury bill rate. As the two assets involved are reasonably close substitutes, one would expect these rates to move together. The specification employed is somewhat ad hoc:¹⁶

$$\begin{aligned}
 (10.11) \quad RCP = & .4513 + .7509 RTB + .3380 RTB(t-1) - .2248 DUCD \\
 & (7.44) \quad (15.5) \quad (7.00) \quad (4.14) \\
 & + .4985 DQ \\
 & (4.88)
 \end{aligned}$$

$$\begin{aligned}
 R^2 &= .991 \\
 SE &= .1548 \\
 DW &= 1.12
 \end{aligned}$$

Once again we have a good fit coupled with a poor Durbin-Watson. A maintained increase in RTB by one percentage point will raise RCP by 1.09 percentage points. This is as we would expect; RCP is the longer of the two rates.

The two dummy variables need some explanation. The invention of certificates of deposit in the early 1960's offered the banks a new way of meeting their short-term liquidity obligations. In theory, the introduction of this new instrument could have had a variety of effects on the structure of interest rates, depending on what portfolio adjustments households and firms decided to make and on how banks elected to respond to these actions. The negative and highly significant coefficient on DUCD indicates that, with the

introduction of a highly attractive alternative, funds were diverted from the Treasury bill market into the commercial paper market, thereby lowering RCP relative to RTB.

The second dummy variable is a bit more straightforward. In 1969 and 1970 banks had difficulty selling certificates of deposit because the maximum interest rate they could legally pay was below what potential customers could earn elsewhere.¹⁷ Banks which were owned by holding companies got around the consequent shortage of funds by selling commercial paper through their parent firm. This increase in borrowing from the commercial paper market served to raise RCP.

10.3 SIMULATION RESULTS

The financial sector consists of identities (10.1) and (10.2) and stochastic equations (10.3) - (10.11). The variable MDD, defined by (10.1) was introduced merely to simplify presentation, so no simulation results for this quantity are presented.

The results of a dynamic simulation of this sector over the sample period 1954 I - 1969 IV are summarized in Table 10.2. Equations (10.1) - (10.11) can be described as "highly simultaneous", in that each endogenous variable appears in at least two equations. Hence here, as with the entire model in Chapter 12, it is theoretically difficult to associate success or failure in tracking a given variable with the strength or weakness of any particular equation.

Still, some results shown in Table 10.2 can be used to illustrate

Table 10.2

Financial Sector Simulation Results:1954 I - 1969 IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
RESR	20.48	.36	1.58	-.092	-2.12**
RESF	-.003	.36	273.	.092	2.12**
RCB	4.47	.40	8.96	.19	4.30***
RCP	3.98	.72	31.60	-.10	-1.14
RCL	5.15	.41	7.36	.16	3.49***
RTD	3.02	.12	3.12	.039	2.71**
RTB	3.34	.74	58.80	-.10	-1.10
MDDP	121.95	2.57	1.88	.10	.31
MTD	104.67	7.01	5.25	-3.66	-4.86***
MCL	52.74	3.11	6.13	-1.24	-3.45***

** Significant at 5%

*** Significant at 1%

a few of the causal aspects of the model. The two volatile rates, RCP and RTB were forecast with large RMS percentage errors, though with little bias. Generally, these rates were over-estimated early in the period and under-estimated later, leading, via the various distributed lag mechanisms, to significant upward biases over the period as a whole in the other three rates.

The upward bias in RTD and the slight downward bias in RTB canceled each other so that demand deposits were forecast with negligible bias. On the other hand, the net upward bias in (RTB+RCB) overshadowed the upward bias in RTD and led to badly biased estimates of time deposits. The over-estimate of RCL "explains" the under-estimation of MCL.

Deposits at commercial banks were on balance understated, leading to a downward bias in free reserves.¹⁸

A comparison of Table 10.2 with Table 10.3, which shows the results for a simulation over the four quarters of 1970, yields few striking conclusions. RCP and RCB are forecast with smaller average percentage errors in the shorter simulation, but they are clearly biased there, while in the longer run they are not. The upward bias in RTD and the fact that it was forecast with less accuracy in the shorter run clearly points to a weakness in equation (10.7).

Examination of the coefficients of equation (10.8) suggests that the combination of the biases in RTD and in RTB should have led to an over-estimate of MDDP. The fact that this variable was forecast

Table 10.3

Financial Sector Simulation Results:1970 I - 1970 IV

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
RESR	28.36	.99	3.43	-.84	-2.86*
RESF	-.52	.99	440.	.84	2.86*
RCB	8.04	.22	2.72	-.14	-1.48
RCP	7.71	1.34	17.72	-1.28	-5.85***
RCL	8.43	.16	1.84	-.025	-.28
RTD	4.74	.17	3.61	.16	4.01**
RTB	6.36	1.45	22.62	-1.40	-6.56***
MDDP	163.24	7.89	4.81	-7.60	-6.14***
MTD	214.80	18.86	8.30	-15.76	-2.63*
MCL	107.62	1.22	1.12	-.13	-.19

* Significant at 10%
 ** Significant at 5%
 *** Significant at 1%

with significant downward bias instead suggests a weakness in (10.8). Similarly, the combination of overestimation of RTD and underestimation of $(RTB+RCB)$ and MTD is suspicious. It is reassuring in a mild sort of way that the downward biases in deposits have the expected impact on the forecasts of required and free reserves.

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Footnotes for Chapter 10

1. For a full description of the procedure employed, see "The X-11 Variant of the Census Method II Seasonal Adjustment Program," U.S. Bureau of the Census, Technical Paper 15, 1965.
2. Chandler [1964, pp. 234-5].
3. Such simulations are presented for an early version of the M.I.T. - F.R.B. - Penn financial sector in de Leeuw and Gramlich [1968, pp. 13-16].
4. Gramlich and Kalchbrenner [1969] present some estimates of the demand for liquid assets which take into account the budget constraint of the non-bank public. Even in this limited investigation, sophisticated assumptions and estimation techniques were required to yield sensible results.
5. The presentation of estimation results in this text is explained below equation (3.22).
6. Natural logarithms are used throughout this text. For linear versions of this specification, see Hendershott [1968] and de Leeuw and Gramlich [1968].
7. See the discussion below equation (6.2).
8. See de Leeuw [1965, 1969], de Leeuw and Gramlich [1968], and Hendershott [1968].
9. Hendershott [1968] presents a much more elegant specification. All sensible variants of his approach, however, produced

completely unsatisfactory estimates with our data.

10. For an interesting exploration of the determinants of the demand for means of payment (i.e., currency and demand deposits, in most developed economies) see Tobin [1956].
11. The basic work on free reserve demand is that of Meigs [1962]. This equation resembles those in de Leeuw and Gramlich [1968], de Leeuw [1969], and Modigliani, Rasche, and Cooper [1970]. See the last reference for a very careful derivation of this specification.
12. In theory, similar changes should arise through fluctuations in time deposits. See Modigliani, Rasche, and Cooper [1970]. We found no evidence that changes in this variable influenced free reserves, which suggests that it is anticipated and/or fully offset by the banking system within a quarter.
13. See Hicks [1939, Ch. 11], and Lutz [1940].
14. See Keynes [1930, Vol. II, p. 135], and Hicks [1939, pp. 146-7].
15. On this approach, which is virtually universal, see Modigliani and Sutch [1966] and the references cited therein.
16. This specification is based on de Leeuw and Gramlich [1968]: it is identical to that used in more recent versions of the M.I.T. - F.R.B. - Penn model. Equations like (10.11), which capture empirical regularities and make no real pretense of modeling economic behavior, are present in all macroeconometric models. We do not know enough to avoid their use entirely.

All one can do is hope that these theoretically weak equations are not of central importance in the final model, though in a system of simultaneous equations it is usually rather difficult to say a priori which are the unimportant equations.

17. In fact, for much of this period the secondary market rate on certificates of deposit was above the legal ceiling. This means that someone wishing to buy such a certificate could obtain it at a lower price from an investor who had originally purchased it from a bank than directly from a bank. Needless to say, banks found it hard to sell CD's under these circumstances.
18. The percentage errors in RESF are essentially meaningless. Errors in this quantity are equal, via (10.2) to minus those in RESR, which is considerably larger. Further, RESF takes on negative values during 32 quarters in the 64 quarters of the sample period. In 58 quarters the forecast value of RESF had the correct sign, a fairly heartening result.

CHAPTER 11

GOVERNMENT DEMAND, FISCAL POLICY,
AND BUDGETARY ACCOUNTS

Quarterly data only permits a condensed portrayal of government expenditures by level of government or by type of expenditure. Hence our model's government demand sector, in which government demand for goods and services is determined, is quite rudimentary.

We shall begin by presenting the National Income and Product Accounts' picture of government activity, including government receipts as well as expenditures, and relating it to our model. This discussion should give the reader some notion of how fiscal policy operates within the present model. The next section presents and discusses the data series employed in this sector. We then discuss the equations making up the government sector and some simulation experiments.

11.1 GOVERNMENT BUDGETARY ACCOUNTS

Table 11.1 exhibits the structure of the government sector in the National Income Accounts, along with the values of the various components in 1969. This table also gives the symbols used in this text for the series involved.

Equations for the four major receipt categories were presented in Chapter 7. The various levels of government set a number of tax rates which, along with activity in the private sector, determine actual receipts. The equations for GRPTX and BCPT, (7.21) and (7.13),

Table 11.1

General Government Receipts and Expenditures: 1969.Billions of Current Dollars*Receipts

Personal Tax and Non-Tax Payments (GRPTX)		117.3
Federal	95.9	
State & Local	21.4	
Corporate Profits Tax Liabilities (BCPT)		42.7
Federal	39.2	
State & Local	3.5	
Indirect Business Tax and Non-Tax Liabilities (BIBT)		85.2
Federal	19.1	
State & Local	66.1	
Contributions for Social Insurance (GRFICA)		53.6
Federal	46.5	
State & Local	7.1	
<u>Total Net Receipts</u>		<u>298.7</u>
Federal	200.7	
State & Local	98.1	

Expenditures

Compensation of Govt. Employees (YGGPI)		103.6
Federal: Defense	32.1	
Federal: Other	10.0	
State & Local	61.5	
Govt. Purchases from the Private Sector (GGDI)		108.6
Federal: Defense	46.7	
Federal: Other	12.6	
State & Local	49.3	
Govt. Transfer Payments to Persons (GETRFP)		61.5
Federal	50.0	
State & Local	11.5	
Net Federal Govt. Transfer Payments to Foreigners (GETFF)		2.1

(Table 11.1, Continued)

Interest Paid by Govt. (GEINT)		13.2
Federal	13.1	
State & Local	.1	
Subsidies Less Current Surplus of Govt. Enterprises (GRSUB)		1.0
Federal	4.6	
State & Local	-3.6	
Less: Federal Govt. Wage Accruals Less Disbursements (GWALD)		(0.0)
<u>Total Net Expenditures</u>		<u>290.0</u>
Federal	171.2	
State & Local	118.8	
<u>Govt. Surplus on National Income Accounts Basis (GSRP)</u>		<u>8.7</u>
Federal	9.3	
State & Local	-.6	

* Source: Survey of Current Business, July 1970; Tables 13 and 14, page 11, and Table 3.11, page 35.

Totals may not add exactly because of rounding error.

contain two such rates, CGBTRT and BCPTRT. No tax rate is present for BIBT, but it is easy to see how a general change in rates would change the corresponding equation, (7.4).

Contributions for Social Insurance are affected by a number of policy parameters. Three of these (TXRSS, TMXEPE, and TXRUB) are employed in the equation for this variable, (7.18). The model thus contains explicitly five tax rates which can be altered in fiscal policy experiments.¹

Let us now turn to government expenditures. The first such item in Table 11.1, compensation of government employees, is treated in the National Income Accounts as government's entire contribution to national product. It is called Gross Government Product. Since the outputs of government activities are seldom sold and thus valued by market prices, they can only be valued in the Accounts at the cost of inputs employed. The second category of government expenditures consists of purchases from private sector businesses. The sum of these two categories is simply called Government Purchases of Goods and Services, and it appears on the product side of the National Income Accounts. Since government purchases can obviously be varied for reasons of fiscal policy, we shall examine their impact in the next chapter.

Net Federal Transfer Payments to Foreigners do not affect Gross National Product or National Income, though they do affect Balance of Payments measures and, of course, government expenditures

on National Income Account. This variable is treated as exogenous in this model.

Transfer payments to Persons appears on the income side of the National Accounts. Here as in the case of taxes, the governments involved determine rates, and actual payments are a function both of rates and of private activity. The only rate appearing explicitly in the equation for GETRFP, (7.17), is TVMOA, though changes in the benefit structure of unemployment insurance are easily incorporated in simulation.

Net interest paid by government, which appears on the income side of the National Accounts as a special class of transfer payment, is not subject to government control in this model. It depends on interest rates, determined in the financial sector, and on the level of interest-bearing government debt, taken as exogenous to the model. In reality, the total amount of outstanding government debt is determined by current and past government receipts and expenditures, and the division between interest-bearing and non-interest-bearing debt is determined by monetary policy. There is thus a government budget constraint, which may have important implications for stabilization policy.² But the relevant receipts and expenditures are not those measured in the national income accounts; to build this constraint into our model we would need to consider alternative measures of government receipts and expenditures.³

The final category of expenditures, net subsidies of government

enterprises, is also considered a transfer payment, and it appears on the income side of the National Accounts. This quantity is taken as exogenous.

This model thus contains a number of tax and expenditure rates which can be varied for purposes of fiscal policy. Also, government compensation of employees and purchases from the private sector can be changed for stabilization reasons. A wide variety of fiscal policy experiments can be performed with this model, some of which will be investigated in Chapter 12.

11.2 THE DATA

The variables appearing in this chapter are shown in Table 11.2. Government purchases from the private sector is defined as total government purchases of goods and services minus government compensation of employees. Given total government purchases of goods and services in current and constant dollars from the National Income Accounts, along with YGGP and YGGPI from the same source, GGD and GGDI are obtained by subtraction.⁴ The implicit deflator PGGD is then computed as $(100 \text{ GGDI} / \text{GGD})$. GSRP is taken directly from the Accounts.

WRPVT is discussed in Chapter 9. Briefly, it is calculated by dividing private compensation of employees (YPRCE) by an unpublished Bureau of Labor Statistics series for total man-hours paid for in the private economy (LPEHR). PGGP is taken directly from the National Income Accounts, as are the remaining endogenous

Table 11.2

Variables Appearing in Chapter 11
and in the Government Demand Sector

(Endogenous Variables - Discussed in Chapter 11)

GGD	Real Government Purchases from the Private Sector (billions of constant dollars, SAAR)
YGGPI	Gross Government Product (billions of current dollars, SAAR).
YGGP	Real Gross Government Product (billions of constant dollars, SAAR).
GSRP	Government Surplus on National Income Account (billions of current dollars, SAAR).

(Endogenous Variables - Determined Elsewhere in the Model)

PGGD	Implicit Price Deflator for Government Purchases from the Private Sector (1957-59 = 100).
WRPVT	Private Sector Average Wage Rate (current dollars per hour, SA).
PGGP	Implicit Price Deflator for Gross Government Product (1957-59 = 100).
GRPTX	Personal Tax and Non-Tax Payments (billions of current dollars, SAAR).
BCPT	Corporate Profits Tax Liability (billions of current dollars, SARR).
BIBT	Indirect Business Tax and Non-Tax Liability (billions of current dollars, SARR).
GRFICA	Contributions for Social Insurance (billions of current dollars, SARR).
GETRFP	Government Transfer Payments to Persons (billions of current dollars, SAAR).

(Table 11.2, Continued)

GEINT Interest Paid by Government (billions of current dollars, SAAR).

(Exogenous Variables)

LEGC Government Civilian Employment (billions of persons).

LEGM Government Military Employment (billions of persons).

GGDI Government Purchases from the Private Sector (billions of current dollars, SAAR).

GETFF Net Federal Transfer Payments to Foreigners (billions of current dollars, SAAR).

GRSUB Subsidies less Current Surplus of Government Enterprises (billions of current dollars, SAAR).

GWALD Federal Government Wage Accruals Less Disbursements (billions of current dollars, SAAR).

TIME Time Trend, Equals 1.0 in 1954I and Rises by 1.0 Each Quarter.

SA = Seasonally-Adjusted.

SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual Rates.

variables determined elsewhere in the model.

LEGM and LEGC are quarterly averages of monthly figures from Employment and Earnings. The computation of GGDI is outlined above, and the last three exogenous variables are taken directly from the Accounts.

11.3 ESTIMATION AND SIMULATION

We take current dollar government purchases from the private sector, GGDI, as exogenous. If a finer breakdown of the government expenditure accounts were available on a quarterly basis, some components of this total could probably be explained within the model.⁵

Given GGDI, GGD is determined by the identity

$$(11.1) \quad GGD = GGDI (100./PGGD),$$

with PGGD determined in the wage-price sector. Similarly, given YGGPI, YGGP is determined by the identity

$$(11.2) \quad YGGP = YGGPI (100./PGGP),$$

and PGGP is also determined in the wage-price sector.

Instead of treating YGGPI, government compensation of employees in current dollars, as exogenous, we take government civilian and military employment as the policy variables, since the wages paid by government are affected and, in the long run, determined by competitive remuneration rates in the private sector.

Define the average government wage rate, WG by

$$(11.3) \quad WG = YGGPI / (LEGC + LEGM).$$

We assume that WG adjusts gradually to WG*, its equilibrium value, according to

$$(11.4) \quad WG = \gamma [WG^* - WG(t-1)] + WG(t-1),$$

a familiar first-order mechanism. We further assume that WG* can be approximated by the following simple linear function of the private sector wage rate, WRPVT, and time:

$$(11.5) \quad WG^* = (a + b \text{ TIME}) \text{ WRPVT}.$$

Substitution of (11.5) into (11.3) and employment of (11.3) leads to the following equation:⁶

$$(11.6) \quad YGGPI = 1608. [WRPVT(LEGM+LEGC)] + 4.952 [WRPVT(LEGM+LEG) \text{ TIME}] \\ (4.54) \qquad \qquad \qquad (3.90) \\ + .5093 [YGGPI(t-1)(LEGM+LEGC)/(LEGM(t-1) + \\ (4.69) \\ LEGC(t-1))]]$$

$$\begin{aligned} R^2 &= .997 \\ SE &= 1.200 \\ DW &= 2.19 \end{aligned}$$

A variety of attempts were made to allow for the differing levels of and trends in government civilian and military wage rates, but none were successful.

In spite of its simplicity, equation (11.6) fits the sample period data quite well. The standard error of \$1.2 billion is small relative to the sample mean of YGGPI of \$58.2 billion. The mean lag of just over a quarter is a bit short, but not totally unreasonable. It should be emphasized, however, that the presence of a trend term in (11.6) means that it cannot safely be used much

beyond the period of fit.

Finally, we have the identity, derived directly from Table 11.1, determining net government surplus:

$$(11.7) \quad \text{GSRP} = (\text{GRPTX} + \text{BCPT} + \text{BIBT} + \text{GRFICA}) \\ - (\text{YGGPI} + \text{GGDI} + \text{GETRFP} + \text{GETFF} + \text{GEINT} + \text{GRSUB}).$$

Of the right-hand side variables, only YGGPI is determined in the government demand sector, while GGDI, GETFF, and GRSUB are exogenous, and the remaining quantities are determined in the income distribution sector, described in Chapter 7.

The government demand sector consists of identities (11.1), (11.2), and (11.7), along with the stochastic equation (11.6). As (11.1) determines GGD as a function of an exogenous quantity (GGDI) and a variable determined outside this sector (PGGD), no simulations of this sector included this equation.

Results of simulations of this sector over the 1954-69 sample period and over the four quarters of 1970 are shown in Table 11.3. Movements in YGGPI and YGGP are tracked quite well; in only two quarters, 1969 III and 1970 IV, did the error in YGGPI exceed \$2.5 billion. The errors in GSRP, identically equal to minus those in YGGPI, are large when taken as percentages of this small quantity, as one would expect.⁷ In any case, there is no evidence of any tendency for this sector to generate biased forecasts.

Table 11.3

Government Demand Sector Simulation Results

<u>Variable</u>	<u>Mean Value</u>	<u>RMS Error</u>	<u>RMS Pct. Error</u>	<u>Mean Error</u>	<u>t-test for Bias</u>
1954 I - 1969 IV					
YGGPI	58.19	1.31	2.17	-.045	-.274
YGGP	47.82	1.03	2.17	-.064	-.496
GSRP	-1.61	1.33	85.04	.015	-.090
1970 I - 1970 IV					
YGGPI	113.30	3.84	3.33	1.26	.601
YGGP	50.70	2.04	3.35	.662	.593
GSRP	-10.08	3.88	45.48	-1.39	-.665

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Footnotes for Chapter 11

1. Since BIBT includes excise taxes, any experiments involving an alteration of the ratio of BIBT to Gross National Product would have to take into account tax shifting and thus consider price changes which would follow any change in excise tax rates. As this is a very complicated business, no such simulations are undertaken. See Fromm and Taubman [1968, ch. 3].
2. See Christ [1967] for a discussion.
3. The series necessary to do this are not available on a quarterly basis. Even if data were no problem, consideration of the government budget constraint would add considerable complexity to our model. On the relations between the various government budget concepts, see J. Scherer [1965] and pp. 69-71 of the 1967 Economic Report of the President.
4. See Section 3.1 for a discussion of the sources of the National Income Accounts series.
5. See Thurow [1969] for an example of what can be done with annual data. See also Ando, Brown, and Adams [1965], Bolton [1969], Gramlich [1969], and Phelps [1969].
6. The description of various parameters associated with estimated equations in this text is described below equation (3.22).

7. The small discrepancies in Table 11.3 between the RMS and mean errors of YGGPI and those of GSRP (and, in 1970, the discrepancy between the percentage errors of YGGPI and YGGP) are due to the failure of the published series to exactly satisfy the theoretical identities. Errors introduced from this source are, however, quite tiny, and no attempt to force the series to satisfy the identities exactly was made.

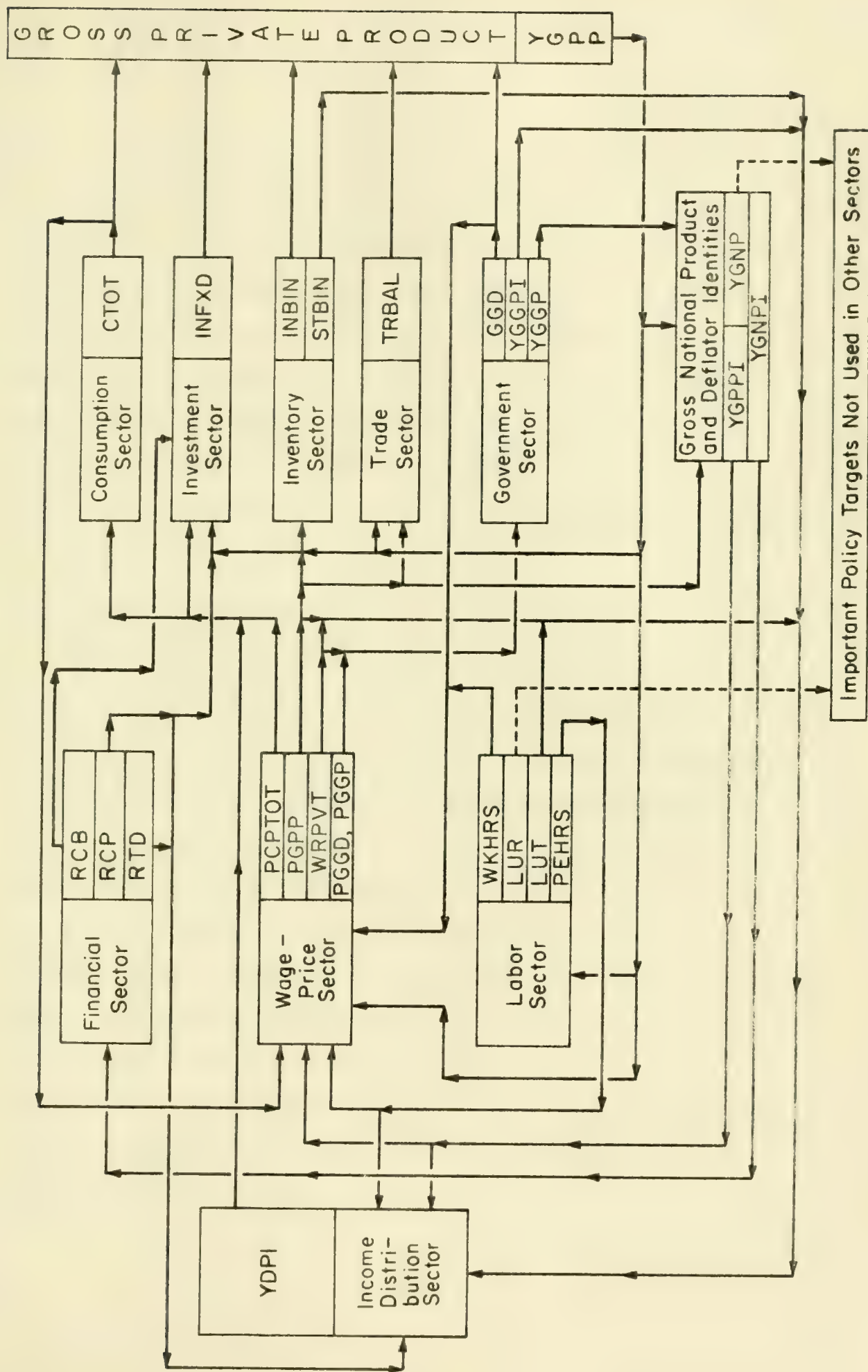
CHAPTER 12

FULL MODEL AND POLICY SIMULATIONS12.1 INTRODUCTION

This chapter has four main parts. The introduction contains a condensed flow diagram of the completed equation system and some brief comments on it. Section 12.2 describes the identities required to close the model and presents a table listing variables which appear therein. The main sections of this chapter examine two sorts of simulations. The first is a long term simulation which puts all the subsystems presented in chapters 3-11 together and for the first time simulates the entire model over the period 1954I to 1969IV; this appears in section 12.3. A critical appraisal of the model's overall performance is made in this context. Section 12.4 presents policy simulations, fourteen altogether, involving different fiscal or monetary strategies that might have been initiated at the onset of the Vietnam War. We choose this period because it clearly represents a mistaken set of economic policies which precipitated the recent inflation. It is not our intent to enter current controversies but to illustrate how econometric models can provide insights into the quantitative effects of different government policies.

The flow diagram in Figure 12.1 has four major parts: final demand, labor and wage-price sector, finance sector and income distribution. It is best to begin from the right side of the chart which

Figure 12.1
Condensed Model Flow Diagram



shows the various components of final demand. Following the arrows back from YGPP to the middle of the chart indicates the crucial impact of YGPP on the labor and price-wage sectors. Above them, the financial sector, through interest rates, influences final demand (YGPP) as well as income distribution. There are many other important linkages reflected by a plethora of lines and arrows which the reader can use to review the more detailed structural links that have been presented in the individual chapters. It should be kept in mind that the model is a system of simultaneous equations, so that in fact all variables are determined together.

12.2 MODEL-CLOSING IDENTITIES

We now present the final equations of our model, five identities which are needed to close it. The variables involved are listed in Table 12.1. All were taken directly from the National Income Accounts, as outlined in Section 3.1, except for GGD, which is the difference between total government purchases of goods and services in constant dollars, from the Accounts, and YGPP.

The first identity determines real gross private product, the key private sector economic activity variable in the model, as the sum of its constant dollar components:

$$(12.1) \quad YGPP = CTOT + INFXD + INBIN + TRBAL + GGD.$$

The variables on the right of this identity were discussed in Chapters 3-6 and 11, respectively. We inflate YGPP to obtain gross private product in current dollars:

Table 12.1

Variables Appearing in Section 12.2
and in the Model-Closing Identities

(Endogenous Variables - Discussed in Section 12.2)

YGPP	Real Gross Private Product (billions of constant dollars, SAAR).
YGPPI	Gross Private Product (billions of current dollars, SAAR).
YGNP	Real Gross National Product (billions of current dollars, SAAR).
YGNPI	Gross National Product (billions of current dollars, SAAR).
PGNP	Implicit Price Deflator for Gross National Product (1957-59 = 100).

(Endogenous Variables - Determined Elsewhere in the Model)

CTOT	Total Consumption Expenditures (billions of constant dollars, SAAR).
INFXD	Gross Private Domestic Fixed Investment (billions of constant dollars, SAAR).
INBIN	Change in Business Inventories (billions of constant dollars, SAAR).
TRBAL	Trade Balance; Equals Net Exports of Goods and Services, Exports minus Imports (billions of constant dollars, SAAR).
GGD	Real Government Purchases from the Private Sector (billions of constant dollars, SAAR).
PGGP	Implicit Price Deflator for Gross Private Product (1957-59 = 100).

YGGP Real Gross Government Product (billions of constant
dollars, SAAR).

YGGPI Gross Government Product (billions of current dollars,
SAAR).

SAAR = Seasonally-Adjusted Quarterly Total Measured at Annual
Rates.

$$(12.2) \quad YGPPI = YGPP (PGPP/100).$$

Gross national product in constant dollars is simply the sum of real gross private product and real gross government product:

$$(12.3) \quad YGNP = YGPP + YGGP,$$

and the corresponding identity holds for the current dollar totals:

$$(12.4) \quad YGNPI = YGPPI + YGGPI,$$

The final quantity of interest, the implicit deflator for gross national product, is obtained from its definition:

$$(12.5) \quad PGNP = 100 (YGNPI/YGNP).$$

The basic national income identities which link the income and product sides are (12.1) and (12.3) which define current dollar private product in terms of output, and (7.2), which defines current dollar private product in terms of income, together with subsidiary income identities (7.6), defining the major constituents of private national income and (7.16), which does the same for personal income. These identities render the model internally consistent in terms of the current national income accounting framework.

12.3 FULL PERIOD SIMULATIONS

Table 12.2 has five main sections; the first relates to aggregate output, the second to prices and wages, the third to employment and hours worked, and the last to major monetary magnitudes.¹ Subtracting the actual values of the indicated variables from the estimates produced by a dynamic simulation of our model over the 1954I-1969IV period yields the model's errors. Their signs and magnitudes

are summarized in Table 12.3, much as was done in Chapters 3-11.² Both absolute and percentage errors are considered. The use of percentage errors helps to interpret the model's forecasting ability, but these statistics lose their meaning when a variable takes on values near or below zero, as occurs, for instance, with inventory investment (INBIN).

The general picture appears in Figure 12.2. The model overpredicts real GNP on the average for the entire period by an average of 2/5 of a percent or \$2.1 billion, with a root mean square error of \$12.5 billions. Errors in the price equations add nothing to this bias, so that the statistics for YGNPI are almost identical to those for YGNP. For a fourteen year simulation covering fifty-six periods, these error magnitudes are a fair representation of what to expect from macromodels using currently available data and theory.

The RMS percentage error of 2% to 2 1/2% in GNP is another way of depicting the state of the art. While a 2% error at current GNP levels amounts to \$20 billion, and would make a short term forecast useless, it provides a reasonable indication of what should be expected of long run economic forecasts.³

Closer examination of the results, best grasped from Figure 12.3, indicates where the errors occurred. The 1955 recovery was seriously underpredicted, a failing common to other economic models which also were unable to foretell the thrust generated by the automobile boom of that year. 1957 was a year of constant output, while the model incorrectly projects large growth in the first quarter and stable

Figure 12.2
Full Period Actual and Simulated YGNP
1954I - 1969IV

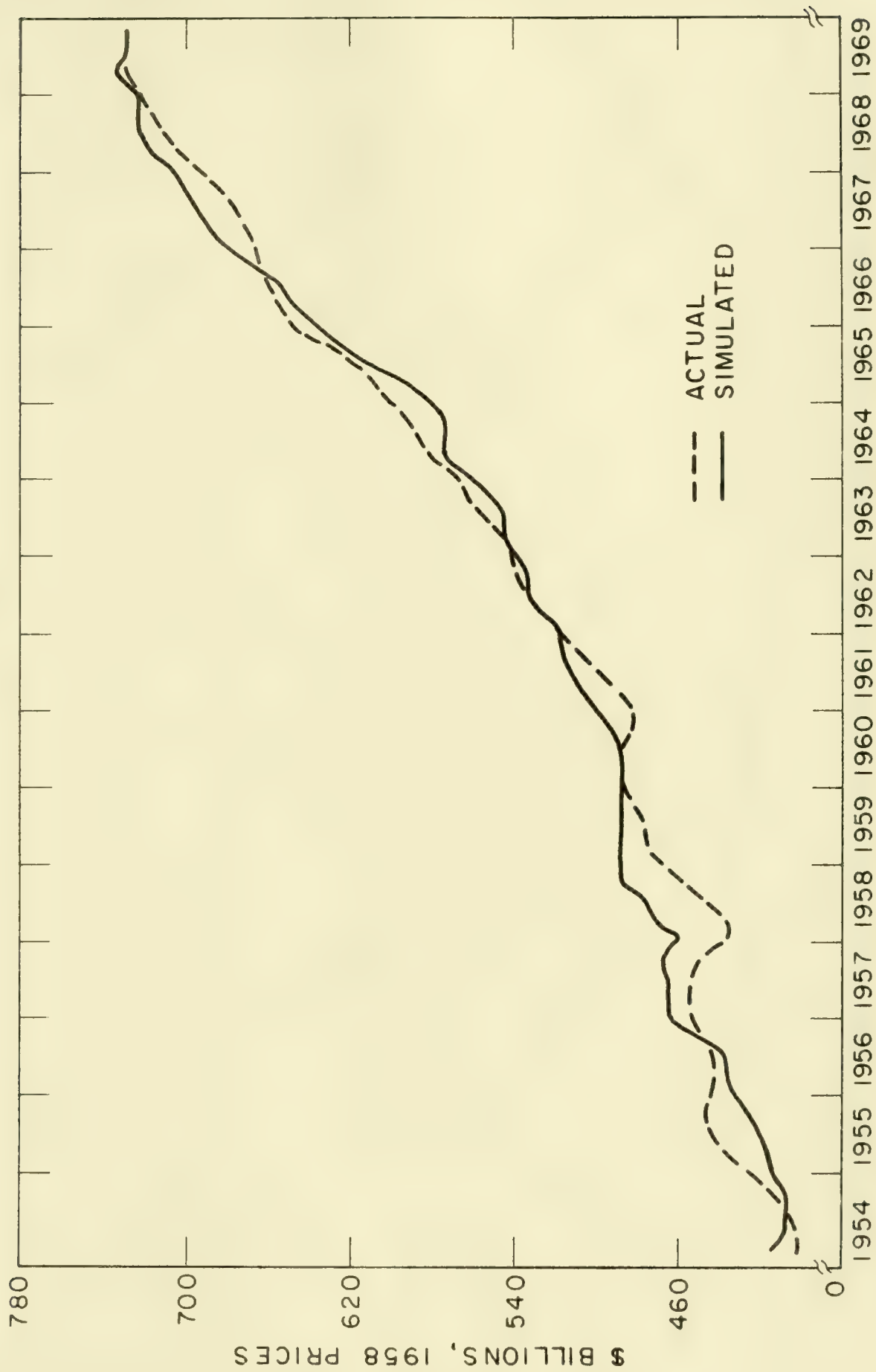
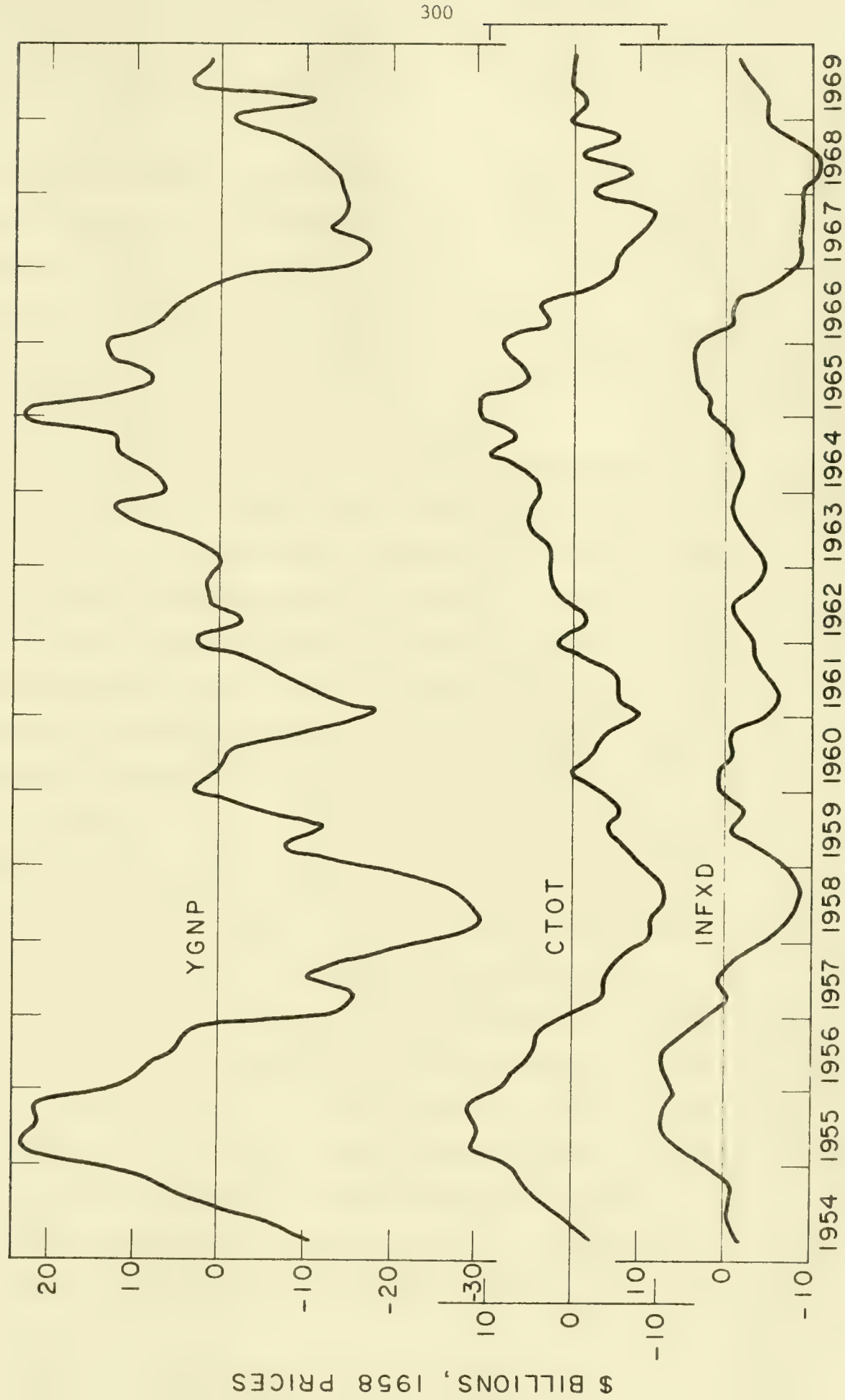


Figure 12.3

Full Period Simulation Errors for YGNP, CTOT and INF XD

1954I - 1969IV



output for the remaining three quarters. The model forecasts a small decline in early 1958 and, apart from a large continuing error, tracked the pace of recovery with some accuracy. Long run simulations are a severe test of an econometric model. Most of them, and this one is no exception, have time paths which portray more inertia than actually exists.

Two much larger econometric models, the Brookings Model and the FRB-MIT model, have comparable error variances. Over the period 1958-1 to 1970-2, the FRB-MIT Model has a RMS error of \$11.3 billion in YGNP, while the Brookings Model has a RMS error of \$10.5 billion for the period 1953III to 1962IV.⁴ (It should be realized that determination of the relative virtues of these different models requires much more extended analysis than that contained in summary statistics for long period simulations.)

The biggest absolute bias in our simulation occurred in fixed investment whose relative error was also the greatest of any major GNP component. Consumption had negligible bias and a small relative error as well. Inventory investment, as usual, had a large RMS error and the largest relative error, although relative errors are less informative for it than the other major GNP components, because it often assumes very small or negative values. Investment in its various forms stands out as the least predictable element in the economy, as detailed results in Chapters 4 and 5 foreshadowed.

Prices and wages had small upward biases. In relative terms, their RMS errors were substantially below those of aggregate output.

In most respects, this sector was effectively modeled. Private compensation had sizable errors though, and we must examine labor demand in order to see where this problem arises. Whereas private employment was tracked with slight bias, the demand for private man hours had relative errors of the same magnitude as GNP itself. It is the combined error in hours and in the demand for labor that builds up to the error in total man hours and thus in private compensation. Thus do errors concatenate throughout the entire system.

Unemployment and the unemployment rate had large relative errors; the latter had an RMS relative error at 18%. This derived magnitude, however important it may be for policy purposes, is always predicted in the long run with large relative errors, because it is the difference between two very similar magnitudes. Independent 1% errors in employment and labor force estimates will produce 40% in forecasts of unemployment when it in turn is 5% of the labor force. Some negative correlation among the errors in the two original magnitudes would bring the average relative error down to the observed vicinity of 20%.

In the monetary sector, the volatile commercial paper and bill rates were poorly predicted on average, although the biases were small. The private long term bond rate and commercial paper rates were better explained. Demand deposits were better and more accurately modeled, although the time deposit forecasts showed bias and large relative error variance. Charts of the errors in the monetary sector confirm our prior belief that it is extremely difficult

1954I - 1969IV

Variable		Level Errors			Percent Errors	
	Mean Value	Mean Error	RMS Error	t-test for Bias	Mean Error	RMS Error
<u>Output and Income</u>						
YGNP	543.676	2.088	12.505	1.34418	0.426	2.538
YGPP	495.850	2.189	12.130	1.45628	0.494	2.707
YGNPI	586.656	5.246	12.480	3.67708***	0.795	2.081
YGPI	528.461	4.954	11.485	3.79488***	0.850	2.178
YDPI	405.435	1.585	5.668	2.3118**	0.397	1.281
YPRCE	283.701	2.641	7.080	3.1911***	0.879	2.469
CTOT	347.920	-0.067	5.888	-0.090447	-0.029	1.831
INBIN	5.108	0.468	3.833	0.976425	19.857	246.177
INFXD	78.043	2.326	5.135	4.03279***	2.839	6.567
<u>Prices and Wages</u>						
PGNP	106.000	0.443	1.536	2.39079**	0.390	1.430
PGPP	104.905	0.420	1.533	2.26111**	0.379	1.454
WRPVT	2.149	0.011	0.036	2.54709**	0.441	1.544
<u>Employment and Hours</u>						
LEPVT	0.062	--	0.001	0.0	0.317	1.465
LPEHR	130.647	0.543	2.703	1.62768	0.442	2.123
LUR	0.049	-0.001	0.009	-0.887412	-0.730	17.887
<u>Monetary</u>						
RTB	3.342	-0.046	0.684	-0.535003	-3.145	50.369
RCB	4.466	0.237	0.472	4.60852***	4.963	10.323
RCL	5.149	0.205	0.461	3.94064***	4.132	8.376
RCP	3.988	-0.042	0.650	-0.513943	-0.996	25.949
MDDP	121.947	0.985	3.515	2.31707**	0.827	2.641
MTD	104.666	-5.559	9.942	-5.35226***	-2.737	7.181

*10% significance level

** 5% significance level

*** 1% significance level

to model financial markets, where so many institutional changes occur and where expectations are of such importance, a judgement based on the extreme amount of serial correlation in the errors as well as their magnitude.

12.4 POLICY EXPERIMENTS

Evaluation of alternative fiscal and monetary policies is one principal application of an econometric model.⁵ Here we illustrate several alternative policies to control inflation that might have been, but were not, pursued at the outset of the Vietnam War. The actual history of this period was roughly as follows. Tax cuts were introduced at the beginning of 1965 to stimulate aggregate demand and thus reduce unemployment. This fiscal action successfully brought unemployment within the year down a full percentage point to 4%, close to prevailing views of what constituted politically acceptable full employment. At this time the Vietnam Military buildup in late 1965 and early 1966 began in deadly earnest, but the Johnson Administration neglected to offset the resultant increase in aggregate demand so that strong inflationary pressures were generated. Belatedly, a 10% personal income tax surcharge was executed in 1968 which expired in 1970. It is against this background that the policy simulations are set.

We proceed as follows. First, the model will be simulated over the period 1966I to 1969IV using the actual exogenous variables and policy parameters for that period to provide a benchmark with which

the policy simulations can be compared. Then changed tax rates and monetary policy parameters will be introduced into the model and the resulting simulation contrasted with the benchmark run. The historical policy parameters and the hypothetical ones will be presented shortly, but first we will examine the benchmark run.

Benchmark Run

The benchmark run summary in Table 12.3 shows that both real and money GNP were overestimated by an average of 1.4%, or approximately \$10 billion dollars. The relative error variability for real GNP was half that of the full period, but the RMS absolute error was about the same. Our comparatively large negative errors in the GNP price level are consistent with other econometric underestimates of inflation rates in this period; all wage rate predictions were invariably low. Positive but mostly small wage rate errors dominated the earlier simulation; these were undoubtedly caused by the upward biased aggregate demand levels projected by the model.

Interest rates, particularly RCB, were consistently underestimated. One reason is that the financial sector equations are highly interdependent and generally place great weight on lagged values of the endogenous variables, so that an initial error remains embedded in the system for a long time. These low interest rates combined with high levels of demand pushed predicted investment well above its actual level; forecasted investment averaged 5 1/2% above its actual level, a much greater relative bias than occurred in the other major GNP component, CTOT.

Complete Model Benchmark Simulation Results

1966I - 1969IV

Variable		Level Errors			Percent Errors	
	<u>Mean Value</u>	<u>Mean Error</u>	<u>RMS Error</u>	<u>t-test for Bias</u>	<u>Mean Error</u>	<u>RMS Error</u>
<u>Output and Income</u>						
YGNP	691.877	9.607	12.848	4.36151***	1.418	1.897
YGNP	835.055	11.049	19.618	2.63978***	1.424	2.395
CTOT	442.049	7.283	8.697	5.93394***	1.678	2.016
INFXD	97.969	5.305	6.605	5.22159***	5.554	6.956
<u>Prices and Wages</u>						
PGNP	120.447	-0.026	0.951	- .105926	-0.002	0.757
WRPVT	2.889	-0.007	0.038	- .725866	-0.179	1.215
<u>Employment and Hours</u>						
LUR	0.037	-0.005	0.008	-3.10087***	-11.978	21.011
<u>Monetary</u>						
RCP	6.095	-0.219	0.465	-2.06773**	-2.851	6.238
RCB	5.961	-0.249	0.324	-4.65197***	-3.915	5.046
RCL	6.681	-0.241	0.536	-1.94958**	-2.862	6.603

* 10% significance level

** 5% significance level

*** 1% significance level

Policy Runs

We consider, altogether, fourteen policy simulations. Six represent fiscal changes for two different periods, another six present alternative monetary policies, and two combine both sets of policies. Half the simulations refer to policy changes for the two years of the Vietnam buildup - 1966I to 1967IV, while the other half maintain the changes throughout the entire four year period 1966I to 1969IV of inflationary buildup, to find out if prolongation of a policy of restraint was necessary to dampen inflation.

As before, results are summarized in terms of some strategic macroeconomic magnitudes to prevent the reader from being overwhelmed by reams of detailed output. The variables chosen are: real GNP and its two major endogenous components, CTOT and INFXD; the unemployment rate, LUR, the GNP price deflator, PGNP and the private wage rate, WRPVT (these three quantities are the essentials of Phillips curve analysis, i.e., the investigation of an economy's unemployment-inflation tradeoff); and one key interest rate, RCP, the commercial paper rate.

Tables 12.4, 12.5 and 12.6 show the average percentage differences between the Benchmark run and various policy runs for the four year period 1966-1969 inclusive. The percentage difference seemed the single most illuminating measure with which to convey the results. It enables the reader to make comparisons across variables within a policy experiment, as well as across policy runs.

Policy Run	Alternative Policy Parameters	Percent Difference from Benchmark						
		YGNP	CTOT	INFXD	LUR	PGNP	WRPVT	RCP
	Personal Tax Rate Change							
F	Tax rate maintained at 20% over period 1966I to 1967IV (historic mean over same period is 15.10%)	-1.251	-1.972	-2.009	14.707	-0.481	-0.811	-2.469
F	Tax rate maintained at 20% over period 1966I to 1969IV (historic mean over same period is 15.65%)	-2.753	-3.64	-4.097	25.525	-0.438	-1.149	-5.067
	Corporate Tax Rate Change							
F	Tax rate maintained at 55% over period 1966I to 1967IV (historic mean is 41.10%)	-0.447	-0.472	-1.249	4.419	-0.054	-0.16	-0.763
F	Tax rate maintained at 55% over period 1966I to 1969IV (historic mean is 43.70%)	-0.642	-0.609	-1.812	5.662	-0.035	-0.186	-1.108
	Combined Personal and Corporate Tax Rate Changes							
F	Personal tax rate increased to 20% and corporate tax rate increased to 55% over period 1966I to 1967IV	-1.642	-2.377	-3.138	18.648	-0.525	-0.951	-3.149
F	Personal tax rate increased to 20% and corporate tax rate increased to 55% over period 1966I to 1969IV	-3.317	-4.157	-5.753	30.602	-0.467	-1.310	-6.064

Fiscal Policy F1, F3 and F5 are the short period (1966-1967) policy change simulations for, respectively, an increase in the first-bracket personal tax rate from 15% to 20% only,⁶ an increase in the observed corporate tax rate from about 42% to 55% only,⁷ and the combined personal and corporate tax rate increases. The rates chosen for the personal tax rate were especially stiff, since analogous runs with the personal tax rate set at .16 and .18 indicated a negligible impact on major variables including prices.⁸ Not all other policy parameters were pushed so severely in order to restrict the number of results to a manageable quantity. F2, F4 and F6 use the same sequence of policy parameters but for the entire 1966-1969 period.

Certain significant results stand out in Table 12.4. First, even with personal tax rates set at the rigorous level chosen, the maximum reduction in the rate of inflation was half of one percent. While these variables cannot be directly translated into Phillips Curve terms, one can see the sort of "price" one has to pay to achieve the indicated somewhat greater extent of price stability. Take F1 as an example. For reduction in the average rate of inflation of .48%, unemployment must rise by 14.7% and real output must fall by 1.25%.

Second, because of the accelerator mechanism, reductions in consumption spread to investment and the total reduction in real GNP is accomplished by declines of approximately the same proportion in both major GNP components when personal taxes are raised. However, the effect of raising corporate taxes alone, which appears in F3,

has an unbalanced effect, falling most heavily on fixed investment. This disparity arises because the increased corporate tax rate has its primary impact through the cost of capital on investment. The latter responds with a long distributed lag to cost changes. Since investment outlays are a small fraction of total outlays relative to consumption, there is a less than proportional consumption multiplier response to the investment decrease.

Third, it is noticable throughout that the percentage fall in YGNP is less than for either CTOT for INFXD. The reason is that government output, federal military and civilian outlays, as well as state and local government outlays, rose rapidly throughout this period. The purpose of the policy simulations is to evaluate the influence on private demand that various restraints have on the price level. We have assumed that resources available to government remain unchanged, and that private demand is manipulated in order to attain the policy objective of greater price stability.

Fourth, when the short period (F1, F3, F5) experiments are compared to the more ferocious long period simulations (F2, F4, F6), the highly interesting fact emerges that, whereas the rate of inflation is barely touched when the policies of restraint are extended an additional two years (even increasing slightly), the real rate of growth is, as might be expected, significantly worsened. Extended reductions in output and unemployment do have a considerable impact on WRPVT, but unit labor costs fall less (two-thirds as much, approximately). The intervening factor of productivity change explains the

observed behavior. Whereas continued higher levels of unemployment continue to push down wages, unit labor costs fall more slowly, since the benefits of increased productivity associated with more rapidly growing output are foregone in the more stagnant of the two economic environments.

Fifth, the corporate tax policy experiments shown in F3 and F4 have smaller impact than the personal tax experiments, a fact that is of only minor significance as observed above, since it depends on the magnitudes of the changes considered. However, a given increase in unemployment has a smaller effect on the inflation rate when the corporate tax rates is varied than when the personal tax rate is changed. The significance of this is not altogether clear, but it seems likely to depend on the dynamics of wages, prices and productivity, which differ when the relative impact of the changes differ.

Sixth, interest rates do not appear to be highly responsive to the rate of economic activity. The percentage change in RCP is typically twice that in YGNP. When you recall that a 5% difference in an interest rate of 5% is 1/4%, repercussions of fiscal changes in the monetary sector seem mild, in light of typical variations in both output and interest rates. This observation takes on further significance when it is contrasted with monetary policy experiments.⁹

Seventh, the effects of personal tax policy and corporate tax appear to be linearly additive. When the percentage changes in most variables for F1 are added to those for F3, they approximately equal

the impact of the combination of policies simulated in F5. The same remark applies to F2, F4 and the combination, F6, for the longer period.

Monetary Policy We consider two sorts of monetary policies: changes in unborrowed reserves, which are presumed to occur through open market operations, and changes in reserve requirements.¹⁰ The policy chosen to represent open market policy is to assume that unborrowed reserves, RESU, change by one-half the amount that they actually did. It is not sensible to consider once and for all changes in unborrowed reserves as it is for either tax or required reserve parameters, since the Federal Reserve System in fact continuously varies unborrowed reserves in light of actual and anticipated circumstances. Since these policies were not strongly expansionary in 1966 and were contracting in 1969, it makes much more sense to us to modify their policies according to the suggested rule, rather than to conjure up an arbitrary series of numbers after the fact.

The changes in demand deposit reserve requirements, RRQD, from about 15% to 17% and in time deposit reserve requirements, RRQT, from 4% to 5% is more straightforward, and was selected on the basis of evaluating an approximately 10% increase in total reserve requirements, equivalent to a 10% reduction in whatever the (potential) money supply is at each point in time.¹¹ This policy is not strictly comparable to the RESU policy described above, but since the money supply grew over the four year period, both policies will have a net

contractionary effect on money, hence the level of aggregate activity and the rate of inflation.

The main results for the monetary policy simulations shown in Table 12.5 are these. First, the RESU policy in the short run had negligible effects, but in the longer run had much greater effects (compare M1 and M2). The difference between long and short run effect is even greater in the reserve requirement experiments. The reserve requirements policy as formulated had a more pronounced effect on real output than did the open market policy. Neither had a major impact, however, for reasons discussed next.

Second, the effect of monetary restraint is greatest on fixed investment. However investment, including residential construction, plant and equipment (INFXD) is so small relative to consumption, that total income and thence YDPI and CTOT are only slightly influenced. In the historical circumstances, consequent multiplier effects from investment on consumption were in large measure offset by the growth in government demand.

The reader should not be mislead into believing that this implies the impotence of monetary policy within this model, far from it. For one thing, a tighter monetary policy certainly could curb investment sufficiently to offset growth in government expenditures and thus bring into play reductions in income and reduced consumption. Such a policy would, however, be effective only with a longer time delay than a direct tax on personal disposable income. In addition, we are not convinced that our model (or most others for that matter)

Monetary Policy Simulations

Policy Run	Alternative Policy Parameters	Percent Difference from Benchmark						
		YGNP	CTOT	INFXD	LUR	PGNP	WRPVT	RCP
Open Market Operations								
1M	Change in unborrowed reserves decreased to half of historic amount over period 1966I to 1967IV	-0.071	-0.026	-0.441	0.842	-0.005	-0.017	4.481
2M	Change in unborrowed reserves decreased to half of historic amount over period 1966I to 1969IV	-0.181	-0.009	-1.153	1.509	0.015	-0.016	15.559
Reserve Requirement Changes								
3M	Required reserve rates increased 2% for demand deposits and 1% for time deposits during period 1966I to 1967IV (historic means: demand - 14.6% time - 4.29%)	-0.041	0.021	-0.609	1.001	-0.018	-0.019	13.606
4M	Required reserve rates increased 2% for demand deposits and 1% for time deposits during period 1966I to 1969IV (historic means: demand - 14.9% time - 4.26%)	-0.308	0.009	-2.165	2.882	0.021	-0.030	30.958
Combined Open Market Operations and Reserve Requirement Changes								
5M	Unborrowed reserves decreased by half of historic change and required reserve rates +2% for demand deposits and +1% for time deposits over period 1966I to 1967IV	-0.028	0.037	-0.655	1.024	-0.020	-0.019	18.202
6M	Unborrowed reserves half of historic and required reserve rates +2% for demand deposits and +1% for time deposits over period 1966I to 1969IV	-0.434	0.010	-2.975	3.834	0.039	-0.036	45.949

has properly captured the effects of credit rationing on the rate of output, especially residential construction. Evolving financial policies and institutions appear to be lessening the impact of credit rationing so that this phenomenon will be of less importance in the future than it proved to be in 1967, for instance.

Third, the main effect of monetary policy is on monetary magnitudes, interest rates in particular. While this remark is scarcely earthshaking, the policy simulations do inform us that there a strong link exists between the monetary policy instruments and monetary variables, and while the links between monetary and real variables are clearly present, they are not overpowering.¹² To illustrate the point, examine M3 and M4, the stronger of the two sets of monetary policies. A 10% increase in reserve requirements for two years causes a 13.6% increase in RCP and a 31% increase when the policy is in effect for the entire four years. The difference between M3 and M4 is attributable to the more complete response observed in M4 arising from the many long distributed lags in the system. These figures are worth comparing with the fiscal simulations where, for historical values of the monetary policy parameters, interest rates fell only slightly in response to the lower level of activity. In the monetary experiments, on the other hand, substantial differences in interest rates e.g. (see M4) 30% are associated with a difference of 2% in the rate of investment. Whereas the monetary magnitudes differ sharply between the two policy approaches, both are consistent with the view that the markets for real output and for financial

assets do not have great impact on each other.

Fourth, the additive influence of different policies remarked upon in the context of fiscal policy simulations by and large carries over to the monetary policy experiments.

Combined Policies Most of what can be learned within the context of these particular simulations is already contained in the preceeding two discussions, since additivity of policy effects carries over to combinations of fiscal and monetary policy as well as within each separately. Nevertheless, some instructive aspects are present in Table 12.6, whose two rows combine all previous policies, both monetary and fiscal for the two and four year periods of policy change. Fiscal policy effects dominate for reasons that are analytical inconsequential since they depend in an arbitrary way on the particular choice of policy changes.

First, the fiscal policy conclusion that an extended period of restraint is a needlessly expensive way of curbing inflation, indeed is counterproductive, is also true here as a comparison of PGNP in CP1 and CP2 clearly shows.

Second, in these experiments, the monetary policies sharply (how sharply depends on the duration of the parameter change) increase interest rates, but because of fiscal effects on the demand for capital, not by as much as for the fiscal policies alone. That is, the fall in output brought about by higher taxes tends to offset the tendency of tight money to raise interest rates.¹³

Table 12.6

Combined Fiscal and Monetary Policy Simulations

Policy Run	Alternative Policy Parameters	Percent Difference from Benchmark						
		YGNP	CTOT	INFXD	LUR	PGNP	WRPVT	RCP
1C	Personal tax rate at 20%, corporate tax rate at 55%, unborrowed reserves half of historic and required reserve rates +2% for demand deposits and +1% for time deposits, all over period 1966I to 1967IV	-1.704	-2.350	-3.995	20.164	-0.545	-0.979	15.219
2C	Personal tax rate at 20%, corporate tax rate at 55%, unborrowed reserves half of historic and required reserve rates +2% for demand deposits and +1% for time deposits, all over period 1966I to 1969IV	-3.773	-4.150	-8.874	34.806	-0.428	-1.353	40.915

Third, the division of output between CTOT and INFXD reflects the different impacts of the monetary and fiscal policies. A consumer oriented policy (neglecting the important role of residential construction at this juncture) would favor tight money and/or higher corporate profits taxes. A growth and/or profits oriented policy would rely most heavily on personal income taxation as the main restraint to an inflationary threat. These results are in accord with both standard economic theory and our intent to provide an acceptable econometric representation of it. The merit of reasonable econometric work is to provide a quantitative dimension to qualitative analysis, as we have tried to do here.

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FOOTNOTES

1. All of the variables appearing in this table have been defined previously. See Appendix A for descriptions and lists of chapters in which each appears. In all of the simulations reported in this section and the next the disturbances (or errors) of the individual stochastic equations are set to zero. See Nagar [1969] for simulations which treat these quantities as random variables. Our approach is more common and, in terms of computer time, less expensive, though it should be clear that we sacrifice some information about the structure of our model when we ignore the estimated standard deviations of the disturbance terms.
2. See Section 1.3.
3. This simulation was made with the actual values of the exogenous variables. In any real forecasting situation, estimates of future values of these quantities must be used instead; the difficulty of obtaining good forecasts of the required series will then have an important impact on model performance. See Steckler [1966] on this point.
4. For the MIT-FRB model, the calculation was made from Nagar [1969] Table 12.1, Column 4. The figure for the MIT-FRB was obtained in a private communication from Professor Modigliani.
5. For other examples, see Ando and Modigliani [1969], de Leeuw and Gramlich [1968], and Fromm and Taubman [1968]. Our methodology is quite similar to that employed by these authors.
6. See equation (7.21); we have altered CGBTRT.

7. See equation (7.13); we have altered BCPTRT.
8. In a linear model, the impact of a change in any policy variable would be a linear function of the size of the change. In a nonlinear model such as ours this is not true in general. The effects of a policy change depend on initial conditions and nonlinearly on the size of the change. Compare Suits [1962] with Fromm and Taubman [1968].
9. In terms of the standard textbook Hicksian short-period macro model, we have experimented so far with parameters which shift the IS curve. The small observed changes in the interest rate indicate that the economy's LM curve is relatively flat. In order to learn about the slope of the IS curve, in turn, we must examine the impact of monetary policy changes which move the LM curve.
10. See Section 10.1 for a general discussion of monetary policy instruments available to the Federal Reserve.
11. If reserve requirements are 10%, reserves of \$20 billion can support a money supply of $\$20/.10 = \200 billion. If reserve requirements double to 20%, the money supply that can be supported is cut in half. Note that we are implicitly counting time deposits as part of the total money supply. This was done for convenience and not out of any deep feelings about the "correct" definition of the money supply.
12. In terms of the IS-LM model mentioned in footnote 9, these experiments involve shifting the LM curve, and they indicate a

fairly steep IS schedule. For an attempt to develop quantitative estimates of the U.S. economy's IS and LM schedules on the basis of a simple macroeconometric model, see Chow [1967].

13. Again, this is what the standard textbook model would suggest.

Restrictive monetary policy raises the LM curve, while restrictive fiscal policy moves the IS curve to the left, thereby reducing output further, but also lowering the rate of interest.

APPENDIX A

VARIABLES APPEARING IN THE MODEL

This appendix contains alphabetized lists of the 82 endogenous and 35 exogenous variables appearing in the macroeconometric model presented in the text. The chapters in which each variable is presented and discussed are indicated. As in the text, the following abbreviations are employed:

SA = Seasonally Adjusted.

SAAR = Seasonally Adjusted Quarterly Total Measured
at Annual Rates

Endogenous Variables

AVP	Average Value Product of Labor in the Private Sector (current dollars per man-hour, SA); 9.
AVPE	Equilibrium Average Value Product of Labor in the Private Sector (current dollars per man-hour, SA); 9.
BCCA	Capital Consumption Allowances (billions of current dollars, SAAR); 7.
BCIVA	Inventory Valuation Adjustment (billions of current dollars, SAAR); 7.
BCP	Corporate Profits Excluding Inventory Valuation Adjustment (billions of current dollars, SAAR); 7.
BCPI	Corporate Profits and Inventory Valuation Adjustment (billions of current dollars, SAAR); 7.

BCPT	Corporate Profit Tax Liability (billions of current dollars, SAAR); 7, 11.
BDIV	Corporate Dividends (billions of current dollars, SAAR); 7.
BIBT	Indirect Business Tax and Non-Tax Liability (billions of current dollars, SAAR); 7, 11.
BTRF	Business Transfer Payments (billions of current dollars, SAAR); 7.
CDSTS	New Private Non-Farm Housing Starts (billions of constant dollars, SAAR); 4.
CPDUR	Per-Capita Consumption Expenditures on Consumer Durables (constant dollars, SAAR); 3.
CPHS	Per-Capita Consumption Expenditures on Housing Services (constant dollars, SAAR); 3.
CPND	Per-Capita Consumption Expenditures on Non-Durables (constant dollars, SAAR); 3.
CPNS	Per-Capita Consumption Expenditures on Services Other Than Housing Services (constant dollars, SAAR); 3.
CPTOT	Total Consumption Expenditures Per Capita (constant dollars, SAAR); 3.
CTOT	Total Consumption Expenditures (billions of constant dollars, SAAR); 3, 9, 12.
ER	Ratio of Civilian Employment to Civilian Population (SA); 8.
GEINT	Interest Paid by Government (billions of current dollars, SAAR); 7, 11.

GETRFP	Government Transfer Payments to Persons (billions of current dollars, SAAR); 7, 11.
GGD	Real Government Purchases from the Private Sector (billions of current dollars, SAAR); 11, 12.
GRFICA	Contributions for Social Insurance (billions of current dollars, SAAR); 7, 11.
GRPTX	Personal Tax and Non-Tax Payments (billions of current dollars, SAAR); 7, 11.
GSRP	Government Surplus on National Income Accounts Basis (billions of current dollars, SAAR); 11.
INBIN	Change in Business Inventories (billions of constant dollars, SAAR); 5, 12.
INFXD	Gross Private Domestic Fixed Investment (billions of constant dollars, SAAR); 4, 12.
INPL	Gross Private Domestic Investment in Non-Residential Structures (billions of constant dollars, SAAR); 4.
INVEQ	Gross Private Domestic Investment in Producers' Durable Equipment (billions of constant dollars, SAAR); 4.
INVH	Gross Private Domestic Investment in Residential Structures (billions of constant dollars, SAAR); 4.
LCP	Primary Civilian Labor Force (billions of persons, SA); 8.
LCS	Secondary Civilian Labor Force (billions of persons, SA); 8.

LEPVT Total Private Employment (billions of persons, SA); 8.

LET Total Employment (billions of persons, SA); 8.

LPEHR Total Man-Hours Paid For in the Private Sector (billions of hours, SAAR); 7, 8, 9.

LPEHRE Equilibrium Total Man-Hours Paid For in the Private Sector (billions of hours, SAAR); 9.

LUR Unemployment Rate; Unemployment as a Fraction of the Civilian Labor Force, (SA); 8.

LUT Total Unemployment (billions of persons, SA); 7, 8.

LWKHR Average Hours Paid For per Employee per Week in the Private Sector (hours, SA); 8, 9.

MCL Commercial and Industrial Loans at All Commercial Banks (billions of current dollars, SA); 10.

MDD Total Net Demand Deposits (Adjusted) at ALL Commercial Banks (billions of current dollars, SA); 10.

MDDP Net Demand Deposits (Adjusted) other than U.S. Government at All Commercial Banks (billions of current dollars, SA); 10.

MTD Time Deposits at All Commercial Banks (billions of current dollars, SA); 10.

NETINT Net Interest (billions of current dollars, SAAR); 7.

PCTOT	Implicit Price Deflator for Total Consumption Expenditure (1957-59 = 100); 3, 4, 9.
PGGD	Implicit Price Deflator for Government Purchases from the Private Sector (1957-59 = 100); 9, 11.
PGGP	Implicit Price Deflator for Gross Government Product (1957-59 = 100); 9, 11.
PGNP	Implicit Price Deflator for Gross National Product (1957-59 = 100); 12.
PGPP	Implicit Price Deflator for Gross Private Product (1957-59 = 100); 6, 7, 9, 12.
RCB	Average Yield on Aaa-Rated Corporate Bonds (percent per year); 4, 7, 10.
RCL	Average Bank Rate on Short-Term Commercial Loans (percent per year); 10.
RCP	Average Yield on 4-6 Month Prime Commercial Paper (percent per year); 4, 10.
RESF	Free Reserves of Federal Reserve Member Banks (billions of current dollars, SA); 10.
RESR	Required Reserves of Federal Reserve Member Banks (billions of current dollars, SA); 10.
RTB	Average Yield on 3 Month U.S. Treasury Bills (percent); 7, 10.
RTD	Interest Rate on Time Deposits at Commercial Banks (percent per year); 10.
STBIN	Stock of Business Inventories on Hand at the Start

of the Quarter minus the Stock on Hand at the Start of 1947I, Times Four (billions of constant dollars); 5, 7.

STKEQ	Net Stock of Producers' Durable Equipment, End of Quarter (billions of constant dollars); 4.
STKEQT	Transformation of STKEQ (billions of constant dollars); 4.
STKH	Net Stock of Residential Structures, End of Quarter (billions of constant dollars); 4.
STKPL	Net Stock of Non-Residential Structures, End of Quarter (billions of constant dollars); 4.
STKPLT	Transformation of STKPL (billions of constant dollars); 4.
TIM	Imports (billions of constant dollars, SAAR); 6.
TRBAL	Trade Balance; Equals Net Exports of Goods and Services, Exports minus Imports (billions of constant dollars, SAAR); 6, 12.
UCP	User Cost of Non-Residential Structures, Excluding the Impact of Inflation (percent per year); 4.
ULC	Unit Labor Cost in the Private Sector (current dollars per constant dollar of output, SA); 9.
ULCE	Equilibrium Unit Labor Cost in the Private Sector (current dollars per constant dollar of output, SA); 9.

WRPVT	Private Sector Average Wage Rate (current dollars per hour, SA); 7, 9, 11.
Y	Per-Capita Real Disposable Income (constant dollars, SAAR); 3.
YCINT	Interest Paid by Consumers (billions of current dollars, SAAR); 7.
YDPI	Disposable Personal Income (billions of constant dollars, SAAR); 3, 4, 7.
YDPID	Real Disposable Personal Income (billions of constant dollars, SAAR); 4.
YGGP	Real Gross Government Product (billions of constant dollars, SAAR); 11, 12.
YGGPI	Gross Government Product (billions of current dollars, SAAR); 7, 11, 12.
YGNP	Real Gross National Product (billions of constant dollars, SAAR); 12.
YGNPI	Gross National Product (billions of current dollars, SAAR); 10, 12.
YGPP	Real Gross Private Product (billions of constant dollars, SAAR); 4, 5, 6, 8, 9, 12.
YGPPI	Gross Private Product (billions of current dollars, SAAR); 7, 10, 12.
YNIP	Private National Income, Equal to National Income Minus Gross Government Product (billions of current dollars, SAAR); 7.

YPERS	Personal Income (billions of current dollars, SAAR); 7.
YPRCE	Private Compensation of Employees, Equal to Compensation of Employees Minus Gross Government Product (billions of current dollars, SAAR); 7.
YPTOT	Proprietors' Income (billions of current dollars, SAAR); 7.
YRENT	Rental Income of Persons (billions of current dollars, SAAR); 7.

Exogenous Variables

BCPTRT	Observed Corporate Profits Tax Rate (fraction); 4, 7.
CGBTRT	First Bracket Federal Tax Rate on Wages and Salaries (fraction); 7.
DDSTR	Dock Strike Dummy Variable; 6.
DQ	Dummy Variable for 1969 and 1970, Equal to 1.0 Then, Zero Otherwise; 10.
DQ2	Seasonal Dummy Variable, Equal to 1.0 Only in Second Quarters, Zero Otherwise; 4.
DQ3	Seasonal Dummy Variable, Equal to 1.0 Only in Third Quarters, Zero Otherwise; 4.
DQ4	Seasonal Dummy Variable, Equal to 1.0 Only in Fourth Quarters, Zero Otherwise; 4.
DR	Federal Reserve Bank of New York Discount Rate (percent per year); 10.

DUCD	Dummy Variable for the Creation of Certificates of Deposit, Equal to Zero Before 1962 I, One Thereafter; 10.
GDBT	Marketable Interest-Bearing Government Debt Held by Private Investors (billions of current dollars, SA); 7.
GETFF	Net Federal Government Transfer Payments to Foreigners (billions of current dollars, SAAR); 11.
GGDI	Government Purchases from the Private Sector (billions of current dollars, SAAR); 11.
GRSUB	Subsidies less Current Surplus of Government Enterprises (billions of current dollars, SAAR); 7, 11.
GWALD	Federal Government Wage Accruals Less Disbursements (billions of current dollars, SAAR); 11.
LEGC	Government Civilian Employment (billions of persons); 8, 9, 11.
LEGM	Government Military Employment (billions of persons); 8, 9, 11.
LPPOP	Primary Population (billions of persons); 8.
LRPOP	Civilian Population Aged 65 and Over (billions of persons); 7.
LSPOP	Secondary Population (billions of persons); 8.
LTPOP	Total Population (billions of persons); 3, 7.
MDDFG	Federal Government Demand Deposits at All Commercial Banks (billions of current dollars, SA); 10.

PTIM	Implicit Price Deflator for Imports (1957-59 = 100); 6.
RDDEP	Ratio of Deductible Depreciation to Deterioration; 4.
RESU	Unborrowed Reserves of Federal Reserve Member Banks (billions of current dollars, SA); 10.
RRQD	Weighted Average of Required Reserve Ratios Against Demand Deposits of Federal Reserve Member Banks (fraction); 10.
RRQT	Reserve Requirement Ratio Against Time Deposits of Federal Reserve Member Banks (fraction); 10.
RTDMX	Legal Maximum Rate Payable on Commercial Bank Time Deposits Held Six Months or More (percent per year); 10.
STADIS	Statistical Discrepancy (billions of current dollars, SAAR); 7.
TEX	Exports (billions of constant dollars, SAAR); 6.
TIME	Time Trend, Equals 1.0 in 1954I and Rises by 1.0 Each Quarter; 7, 8, 9, 11.
TMXEPE	Index of Maximum OASDHI-Taxable Earnings per Employee (1958-100); 7.
TVMOA	Maximum Per-Family OASDHI Benefits per Month (cur- rent dollars); 7.
TXRUB	Employer's Tax Rate for Unemployment Insurance (percent); 7.

TXRSS Employer and Employee Tax Rate for OASDHI (percent);
7.

WSACCR Wage Accruals Less Disbursements (billions of current dollars, SAAR); 7.

APPENDIX B

THE EQUATIONS OF THE MODEL

In this appendix we list the 42 stochastic equations and 40 identities which make up our model. Beside each equation is the number it was assigned in the chapter of the text where it was presented. We first list the stochastic equations and then the identities; the order within each list is the order of presentation in the text.

Stochastic Equations

$$(3.23) \quad \text{CPND} = 182.821 + .23661[Y - Y(t-1)] + .15339 Y(t-1) \\ + .42661 \text{CPND}(t-1)$$

$$(3.24) \quad \text{CPDUR} = -50.1767 + .28281[Y - Y(t-1)] + .05734 Y(t-1) \\ + .7679 \text{CPDUR}(t-1)$$

$$(3.25) \quad \text{CPNS} = .02432 Y + .89512 \text{CPNS}(t-1)$$

$$(3.26) \quad \text{CPHS}/Y = .00426 + .96943[\text{CPHS}(t-1)/Y(t-1)]$$

$$(4.23) \quad \text{INVH} = 4.22239 + 1.03239[.41 \text{CDSTS} + .49 \text{CDSTS}(t-1) \\ + .10 \text{CDSTS}(t-2)] + .004149996 \text{YDPID} - .03929 \text{DQ2} \\ - .51046 \text{DQ3} - .37553 \text{DQ4}$$

$$(4.25) \quad [\text{CDSTS} - .024 \text{STKH}(t-1)] = .03929[\text{YDPID}(t-1)/\text{RCP}(t-1)]^{.30} \\ - \text{YDPID}(t-2)/\text{RCP}(t-2)]^{.30} + .03792[\text{YDPID}(t-2)/\text{RCP}(t-2)]^{.30} \\ - \text{YDPID}(t-3)/\text{RCP}(t-3)]^{.30} + .96642[\text{CDSTS}(t-1) \\ - .024 \text{STKH}(t-2)]$$

- (4.30) $[INPL - STKPLT] = .12097[.30[YGPP(t-1)/UCP(t-1) \cdot^{.20}$
 $-YGPP(t-2)/UCP(t-2) \cdot^{.20}] + .38[YGPP(t-2)/UCP(t-2) \cdot^{.20}$
 $-YGPP(t-3)/UCP(t-3) \cdot^{.20}] + .18[YGPP(t-3)/UCP(t-3) \cdot^{.20}$
 $-YGPP(t-4)/UCP(t-4) \cdot^{.20}] + .11[YGPP(t-4)/UCP(t-4) \cdot^{.20}$
 $-YGPP(t-5)/UCP(t-5) \cdot^{.20}] + .03[YGPP(t-5)/UCP(t-5) \cdot^{.20}$
 $-YGPP(t-6)/UCP(t-6) \cdot^{.20}]] + .9678[INPL(t-1) - STKPLT(t-1)]$
- (4.35) $[INVEQ - STKEQT] = .23691[.319(YGPP - YGPP(t-1)) + .266(YGPP(t-1)$
 $-YGPP(t-2)) + .102(YGPP(t-2) - YGPP(t-3)) + .032(YGPP(t-3)$
 $-YGPP(t-4)) + .028(YGPP(t-4) - YGPP(t-5)) + .054(YGPP(t-5)$
 $-YGPP(t-6)) + .082(YGPP(t-6) - YGPP(t-7)) + .083(YGPP(t-7)$
 $-YGPP(t-8)) + .033(YGPP(t-8) - YGPP(t-9))]$
 $+ .91538[INVEQ(t-1) - STKEQT(t-1)]$
- (5.15) $INBIN = .28750[YGPP - YGPP(t-1)] + .23009[YGPP(t-1) - YGPP(t-2)]$
 $+ .18402[YGPP(t-2) - YGPP(t-3)] + .14874[YGPP(t-3) - YGPP(t-4)]$
 $+ .12367[YGPP(t-4) - YGPP(t-5)] + .10825[YGPP(t-5) - YGPP(t-6)]$
 $+ .10192[YGPP(t-6) - YGPP(t-7)]$
- (6.2) $\log(TIM) = -1.86526 + .44526 \log(YGPP) - .14212 \log(PTIM/PGPP)$
 $+ .73135 \log(TIM(t-1)) + .06751 \log(1.-DDSTR)$
- (7.3) $BCCA = .006859999 YGPPI + .94386 BCCA(t-1)$
- (7.4) $[BIBT - BIBT(t-1)] = .04062[YGPP - YGPP(t-1)] + .00115 YGPPI$
- (7.5) $BTRF = .0002445 YGPPI + .95709 BTRF(t-1)$
- (7.8) $\log[YPTOT/YGPPI] = -.42903 + .79824 \log[YPTOT(t-1)/YGPP(t-1)]$
 $- .00128 TIME$
- (7.9) $YRENT/YGPPI = .80201[YRENT(t-1)/YGPP(t-1)] + 17.0166$
 $(LTPOP/YGPP)$

$$(7.10) \quad \text{NETINT} = .000153473(\text{RCB YGPPI}) + .97988[\text{RCB NETINT}(t-1)/\text{RCB}(t-1)]$$

$$(7.14) \quad \text{BCIVA} = -.36343 [\text{PGPP}-\text{PGPP}(t-1)] - .00436 [\text{STBIN}(\text{PGPP}-\text{PGPP}(t-1))] + .38559 \text{ BCIVA}(t-1)$$

$$(7.15) \quad \text{BDIV} = .0242(\text{BCP}-\text{BCPT}) + .96313 \text{ BDIV}(t-1)$$

$$(7.16) \quad [\text{GEINT}-\text{GEINT}(t-1)] = .10478 + .00977[\text{RTB}(\text{GDBT}-\text{GDBT}(t-1))] + .00147[\text{GDBT}(\text{RTB}-\text{RTB}(t-1))]$$

$$(7.17) \quad \text{GETRFP} = -4.93575 + 6.5548(\text{LRPOP TVMOA}) + 1012.04 \text{ LUT} + 36.4442(\text{TIME LUT})$$

$$(7.18) \quad \text{GRFICA} = -7.20201 + .00197[\text{TXRSS}(\text{YPRCE}+\text{YPTOT}(\text{TMXEPE}) + .03027(\text{TXRUB YPRCE})$$

$$(7.19) \quad \text{YCINT} = .49239 \text{ RCB} - .20909 (\text{RCB})^2 + .00264 (\text{RCB YDPI}) + .67958[\text{RCB YCINT}(t-1)/\text{RCB}(t-1)]$$

$$(7.21) \quad \log(\text{GRPTX}/\text{LTPOP}) = -5.681 + .5705 \log(\text{CGBTRT}) + 1.587 \log(\text{YPERS}/\text{LTPOP})$$

$$(8.7) \quad \text{LCS}/\text{LSPOP} = -.02275 + .61637(\text{LCS}(t-1)/\text{LSPOP}(t-1)) + .000360799 \text{ TIME} + .028371 \text{ ER}$$

$$(8.8) \quad \text{LCP}/\text{LPPOP} = .28719 + .63521 (\text{LCP}(t-1)/\text{LPPOP}(t-1)) - .000341236 \text{ TIME} + .05973 \text{ ER}$$

$$(8.12) \quad \log(\text{LPEHR}) = -.00237 \text{ TIME} + .32969 \log(\text{YGPP}) + .59727 \log[\text{LPEHR}(t-1)]$$

$$(8.18) \quad \log(\text{LWKHR}) - .59727 \log[\text{LWKHR}(t-1)] = .37795 + .14641 \log(\text{YGPP}) - .00151 \text{ TIME} - .09232 \log[\text{LEPVT}(t-1)]$$

$$(9.6) \quad \log(\text{WRPVT}) = -3.20745 + .17844[\log(\text{AVP}) - \log(\text{AVPE})] \\ + .61084 \log(\text{AVPE}) + .41104 \log[\text{WRPVT}(t-1)] \\ + .78699[\log(\text{PCTOT}) - \log(\text{PCTOT}(t-1))]$$

$$(9.7) \quad \text{PGGP} = 2.60749 + 13.8767 \text{WRPVT} + .75264 \text{PGGP}(t-1) \\ - 2.91318[\text{WRPVT} \text{LEGM}/(\text{LEGM} + \text{LEGC})]$$

$$(9.10) \quad \log(\text{PGPP}) = 1.47246 + .15374[\log(\text{ULC}) - \log(\text{ULCE})] \\ + .27763 \log(\text{ULCE}) + .64894 \log[\text{PGPP}(t-1)] \\ + .08461 \log(\text{LWKHR}) + .00031611 \text{TIME}$$

$$(9.11) \quad [\log(\text{PCTOT}) - \log(\text{PCTOT}(t-1))] = -.04468 + .7902[\log(\text{PGPP}) - \\ \log(\text{PGPP}(t-1))] + .15181[\log(\text{PGPP}(t-1)) - \\ \log(\text{PGPP}(t-2))] + .0634(\text{CTOT}/\text{YGPP})$$

$$(9.12) \quad [\log(\text{PGGD}) - \log(\text{PGGD}(t-1))] = 1.31853[\log(\text{PGPP}) - \log(\text{PGPP}(t-1))]$$

$$(10.3) \quad \text{RESR} = 2.41677 + .78044(\text{RRQD MDD}) + .64875(\text{RRQT MTD})$$

$$(10.4) \quad \text{MCL} = -2.26681 - .70205(\text{RCL} - \text{RTB}) + .01910 \text{YGPP1} \\ + .89719 \text{MCL}(t-1)$$

$$(10.5) \quad \log(\text{RCL}) = .22307 + .08316 \log(\text{RCP}) + .31700 \log(\text{RCB}) \\ + .51118 \log(\text{RCL}(t-1))$$

$$(10.6) \quad \text{MTD}/\text{YGNPI} = .00680 + .00430 \text{RTD} - .00172 (\text{RTB} + \text{RCB}) \\ - .19629[(\text{YGNPI} - \text{YGNPI}(t-1))/\text{YGNPI}(t-1)] \\ + .98778[\text{MTD}(t-1)/\text{YGNPI}(t-1)]$$

$$(10.7) \quad \log(\text{RTD}/\text{RTDMX}) = .03550 \log[(\text{RCB} + \text{DR})/(2 \text{RTDMX})] \\ + .96136 \log[\text{RTD}(t-1)/\text{RTDMX}(t-1)]$$

$$(10.8) \quad \text{MDDP}/\text{YGNPI} = .05500 - .00421 \text{RTD} - .00132 \text{RTB} \\ + .81914[\text{MDDP}(t-1)/\text{YGNPI}(t-1)]$$

$$(10.9) \quad \text{RESF}/\text{MDD}(t-1) = .000364832 + .00200(\text{DR}-\text{RTB}) - .62768[\text{RRQD} \\ [\text{MCL}-\text{MCL}(t-1)]/\text{MDD}(t-1)] + .44716[(1-\text{RRQD})[\text{RESU}- \\ \text{RESU}(t-1)]/\text{MDD}(t-1)] + .66329[\text{RESF}(t-1)/\text{MDD}(t-1)]$$

$$(10.10) \quad \log(\text{RCB}) = .04275 \log(\text{RCP}) + .97060 \log[\text{RCB}(t-1)]$$

$$(10.11) \quad \text{RCP} = .45129 + .75088 \text{RTB} + .33803 \text{RTB}(t-1) + .49849 \text{DQ} \\ - .22484 \text{DUCD}$$

$$(11.6) \quad \text{YGGPI} = 1608.31[\text{WRPVT}(\text{LEGC}+\text{LEGM})] - 4.95242[\text{WRPVT}(\text{LEGM}+\text{LEGC}) \\ \text{TIME}] + .50933[\text{YGGPI}(t-1)(\text{LEGM}+\text{LEGC})/(\text{LEGM}(t-1) \\ + \text{LEGC}(t-1))]$$

Identities

$$(3.19) \quad Y = (\text{YDPI}/\text{LTPOP})(100./\text{PCTOT})$$

$$(3.20) \quad \text{CTOT} = \text{CPTOT LTPOP}$$

$$(3.21) \quad \text{CPTOT} = \text{CPDUR} + \text{CPND} + \text{CPNS} + \text{CPHS}$$

$$(4.1) \quad \text{STKEQ} = .25 \text{INVEQ} + (1.-.148/4.)\text{STKEQ}(t-1)$$

$$(4.2) \quad \text{STKPL} = .25 \text{INPL} + (1.-.0586/4.)\text{STKPL}(t-1)$$

$$(4.3) \quad \text{STKH} = .25 \text{INVH} + (1.-.024/4.)\text{STKH}(t-1)$$

$$(4.7) \quad \text{UCP} = 5.86[1.-(\text{RDDEP})\text{BCPTRT}]/(1.-\text{BCPTRT}) \\ + (2.55 \text{RCB})[1.-(.125)\text{BCPTRT}]/(1.-\text{BCPTRT})$$

$$(4.9) \quad \text{YDPID} = \text{YDPI}(100./\text{PCTOT})$$

$$(4.21) \quad \text{INFXD} = \text{INVH} + \text{INPL} + \text{INVEQ}$$

$$(4.29) \quad \text{STKPLT} = .0586[.30 \text{STKPL}(t-2) + .38 \text{STKPL}(t-3) \\ + .18 \text{STKPL}(t-4) + .11 \text{STKPL}(t-5) + .03 \text{STKPL}(t-6)]$$

- (4.34) $STKEQT = .148 [.319 STKEQ(t-1) + .266 STKEQ(t-2)$
 $+ .102 STKEQ(t-3) + .032 STKEQ(t-4) + .028 STKEQ(t-5)$
 $+ .054 STKEQ(t-6) + .082 STKEQ(t-7) + .083 STKEQ(t-8)$
 $+ .033 STKEQ(t-9)]$
- (5.1) $STBIN = STBIN(t-1) + INBIN(t-1)$
- (6.1) $TRBAL = TEX - TIM$
- (7.2) $YNIP = YGPPI - BCCA - BIBT - BTRF + GRSUB - STADIS$
- (7.6) $BCPI = YNIP - YPRCE - YPTOT - YRENT - NETINT$
- (7.7) $YPRCE = WRPVT LPEHR$
- (7.11) $YPERS = YNIP + YGGPI - BCPI + BDIV - WSACCR + BTRF$
 $+ GEINT + GETRFP - GRFICA + YCINT$
- (7.12) $BCP = BCPI - BCIVA$
- (7.13) $BCPT = BCP BCPTRT$
- (7.20) $YDPI = YPERS - GRPTX$
- (8.2) $LEPVT = LPEHR / (52 LWKHR)$
- (8.3) $LET = LEPVT + LEGC + LEGM$
- (8.4) $LUT = LCP + LCS - (LEPVT + LEGC)$
- (8.5) $LUR = LUT / (LCP + LCS)$
- (8.6) $ER = (LET - LEGM) / (LPPOP + LSPOP - LEGM)$
- (9.2) $\log(LPEHRE) = -.005884836 TIME + .8186378 \log(YGPP)$
- (9.4) $AVP = (PGPP YGPP) / LPEHR$
- (9.5) $AVPE = (PGPP YGPP) / LPEHR$
- (9.8) $ULC = (WRPVT LPEHR) / YGPP$
- (9.9) $ULCE = (WRPVT LPEHRE) / UGPP$

$$(9.9) \quad \text{ULCE} = (\text{WRPVT LPEHRE}) / \text{YGPP}$$

$$(10.1) \quad \text{MDD} = \text{MDDP} + \text{MDDFG}$$

$$(10.2) \quad \text{RESF} = \text{RESU} - \text{RESR}$$

$$(11.1) \quad \text{GGD} = \text{GGDI} (100/\text{PGGD})$$

$$(11.2) \quad YGGP = YGGPI \ (100/PGGP)$$

$$(11.4) \quad \text{GSRP} = (\text{GRPTX} + \text{BCPT} + \text{BIBT} + \text{GRFICA}) - (\text{YGGPI} + \text{GGDI} + \text{GETRFP} + \text{GETFF} + \text{GEINT} + \text{GRSUB} - \text{GWALD})$$

$$(12.1) \quad YGPP = CTOT + INF XD + INBIN + TRBAL + GGD$$

$$(12.2) \quad YGPI = YGPP(PGPP/100)$$

$$(12.3) \quad YGNP = YGPP + YGGP$$

$$(12.4) \quad \text{YGNPI} = \text{YGPPI} + \text{YGGPI}$$

$$(12.5) \quad \text{PGNP} = 100(\text{YGNPI}/\text{YGNP})$$

BASEMENT Date Due

DEC 16 '75
 JAN 13 '76
 FEB 12 '76
 MAR 16 '76
 APR 22 '76
 AUG 12 '79
~~Sept 10 '76~~
~~OCT 08 '76~~
 NOV 26 '76
 FEB 02 '77
 MAR 02 '77
 APR 03 '77
 APR 14 '77
 JUN 07 '77

APR 06 '80

Lib-26-67

